A GENERAL MULTIRESOLUTION APPROACH TO THE ESTIMATION OF DENSE FIELDS IN REMOTE SENSING

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ABSTRACT

A fast multiscale optimal interpolation algorithm has been adapted to the mapping of hydrographic and other oceanographic data. This multiscale algorithm produces solution and error estimates which are consistent with those obtained from exact least-squares methods, but at a small fraction of the computational cost. Problems whose solution would be completely impractical using exact least-squares, that is problems with tens or hundreds of thousands of measurements and estimation grid points, can easily be solved on a small workstation using the multiscale algorithm. Contrary to methods previously proposed for solving large least-squares problems, the multiscale approach provides error statistics while permitting long-range correlations, using all measurements, and permitting arbitrary measurement locations.

A set of MATLAB-callable routines which implements the multiscale algorithm and reproduces the results obtained in this paper are available by anonymous FTP; see the last section of this paper for details.

1. INTRODUCTION

The production of maps — dense, regularly gridded sets of estimates — has become a problem of tremendous interest to scientific researchers working in remote sensing. Such maps are appealing not only for aesthetic reasons but also serve a scientific purpose: remote sensing measurements, for example those taken from a ship, are often rather sparsely sampled; such sparse fields are difficult for scientists to interpret, and are inconvenient or inappropriate for driving simulation programs.

The challenge in the production of such maps is the large number of estimates required ($\gg 10000$), the sparse and irregular nature of the supplied measurements, and the requirement that estimation error statistics be provided. Many oceanographers continue to solve least-square problems by matrix inversion, however this approach becomes completely impractical for large two-dimensional estimation problems. Instead, we propose to address such problems using a multiscale estimation framework which easily handles such estimation problems, and furthermore gives the user explicit control over the tradeoff between statistical fidelity and computational effort.

We will be applying our method to a problem of current scientific interest: the estimation of oceanic temperature maps from irregular measurements (small dots in Figure 1). We stack the random field into a large random vector x, having prior statistics

$$E[x] = 0, \quad \text{cov}(x) = S \qquad (1)$$

where S is a smooth correlation function (isotropic Gaussian) determined via empirical means.

2. MULTISCALE APPROACH

To be sure, methods have been proposed to solve large estimation problems while avoiding brute-force matrix

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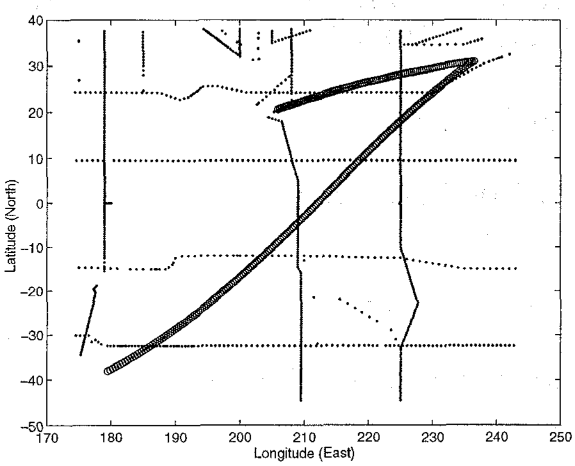

Figure 1: Sample problem of interest: ship-based temperature measurements are taken at each location marked by a dot. We are interested in determining a dense map of temperature, and validating our estimates on the thick line.

inversion, however each of these methods has some limitation which we find to be too constraining:

- FFT methods require that the measurements be regularly sampled in space, and that the measurement noise process be stationary.
- Local methods base each estimate upon only those measurements in its local vicinity, ignoring the rest.
- Subset methods base each estimate upon some (random or deterministic) subset of the measurements, ignoring the rest.
- Hierarchical methods such as multigrid do not provide estimation error statistics.

In this section we present a method which permits the rapid solution of least-squares problems while maintaining long-range correlations, using all measurements, and permitting arbitrary measurement locations. This method is based on stochastic processes modeled on trees; that is, processes z such that

$$z(s) = A(s)z(s\bar{\gamma}) + B(s)w(s) \quad (2)$$
$$y(s) = C(s)z(s) + v(s) \quad (3)$$

where s indexes the nodes of a tree, $s\bar{\gamma}$ represents the parent node of s, $w(s)$ is a white-noise process, and $y(s)$ represents the multiscale measurement process. We select a quad-tree (illustrated in Figure 2) as our tree structure; that is, each node above the finest scale

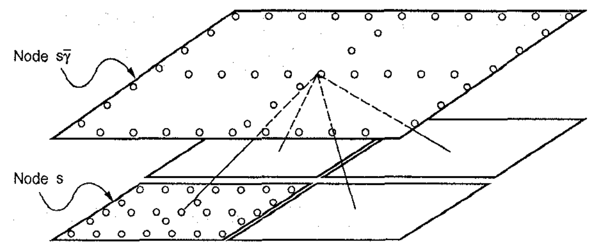

Figure 2: An illustration of the definition of the state at each multiscale tree node: the state at each node s is made up pixels (small circles) sampled along the boundaries of node s and its four children.

has four descendents, each descendent representing one quadrant of the region represented by the parent. For the purposes of this paper, we will focus primarily upon the finest scale of the tree: our hydrographic process x of interest will live on the finest scale of the tree, and all of the ship-based measurements will appear on finest scale nodes.

For multiscale models of the form (2),(3) there exist very fast estimation algorithms[1] which compute the estimates $\hat{z}(s)$ and estimation error covariances $\tilde{P}(s)$ at each tree node s. The challenge in using these models stems from the need to select appropriate matrices A, B such that the nodes on the finest scale of the tree possess the desired statistical covariance S. We propose to develop multiscale models motivated by the method of canonical correlations[2, 3], based on the following observation: the statistical role of $z(s)$ in (2) is to mutually decorrelate the trees descending from node s and to decorrelate these from the remainder of the tree. We select the state at each node s to equal a subset of the process x sampled along the perimeter of s and its children (as illustrated in Figure 2). That is,

$$z(s) = W(s)x \quad (4)$$

where $W(s)$ is a matrix, sampling x along the perimeter of s and the children of s. Once $W(s)$ has been chosen for each tree node, the multiscale model follows immediately:

$$A(s) = \left[W(s)SW^T(s\bar{\gamma})\right]\left[W(s\bar{\gamma})SW^T(s\bar{\gamma})\right]^{-1} \quad (5)$$
$$BB^T(s) = \text{cov}\,(z(s)) - \text{cov}\,(A(s)z(s\bar{\gamma})) \quad (6)$$

This class of multiscale models leads to the following tradeoff, under explicit control of the user: the more densely that $W(s)$ samples x along the perimeter of s and its children (i.e., the higher the dimension of $z(s)$) the greater the statistical fidelity of the resulting estimates, but the greater the computational burden. Thus in the event that the prior statistics S were rather

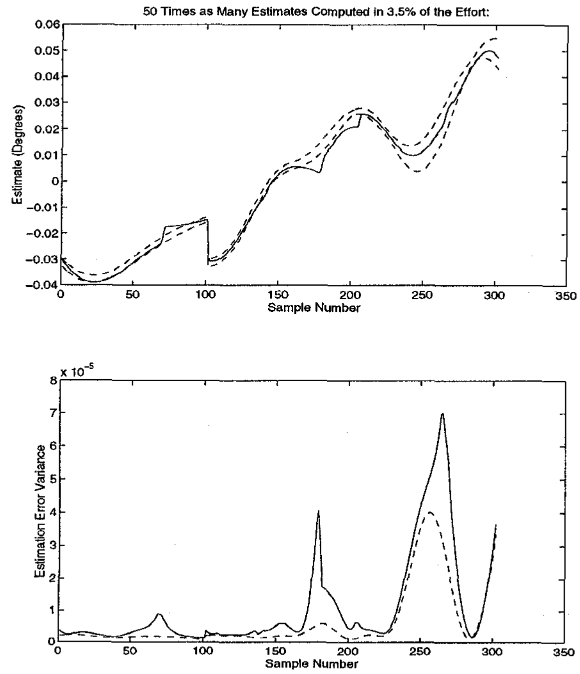

Figure 3: A comparison of multiscale and exact least-squares results along the thick line of Figure 1. The top panel shows the multiscale estimates (solid), bounded by the exact estimates $\pm$ one standard deviation (dashed). The bottom panel compares estimation error standard deviations.

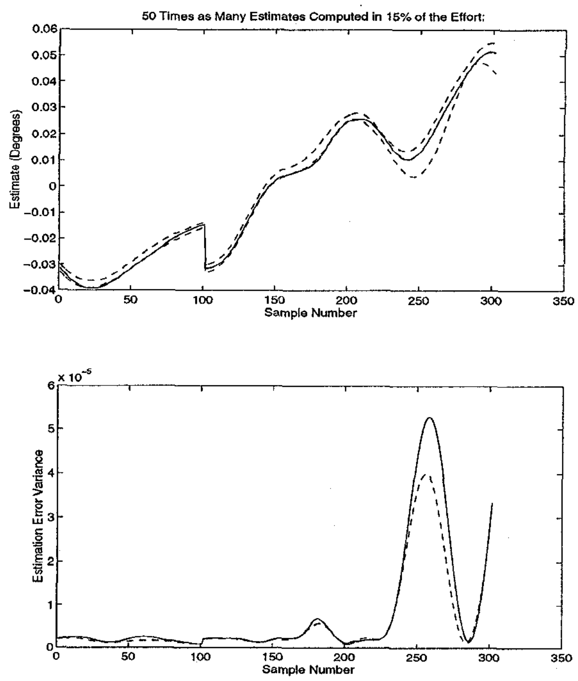

Figure 4: Same as Figure 3, but using a multiscale model representing four times the computational effort, but still computing 50 times as many estimates in 15% of the time for matrix inversion. The multiscale estimates now lie neatly within the one-sigma envelope of the exact solution.

approximate, the user might opt for a relatively small state dimension for rapid estimation.

3. EXPERIMENTAL RESULTS

The successful development of a dense map estimator offers many exciting scientific possibilities, of which only a tiny sample can be illustrated here.

Figure 3 and 4 show two sets of estimation results, based on low and high order multiscale models, respectively. These figures compare the multiscale estimates and error statistics with the *exact* least-squares results along the thick path shown in Figure 1. The multiscale estimates of Figure 3, based upon a sparse state sampling in $W(s)$, are similar to the exact solution, however the significant difference between the exact and multiscale estimation error statistics suggests that these results might be inadequate for a detailed model validation study. On the other hand the multiscale estimates and the estimation error variances in Figure 4, based on a denser state sampling in $W(s)$, are quite close to the exact solution. These results required about four times the effort of that for Figure 3, however this effort is still a tiny fraction of the effort to compute the exact result via matrix inversion. Furthermore, this work was motivated from the start by the fact that the prior statistics S are somewhat uncertain; thus the small differences between the estimates produced by our multiscale method and those of the exact approach are essentially inconsequential, although the difference in computational effort is staggering.

The dense fields corresponding to the multiscale model of Figure 4 are shown in Figure 5. Clearly no comparison with exact least-squares methods are possible, because an estimation problem of this magnitude (16400 estimates) is well beyond the practical limit of matrix inversion. Furthermore, despite the fact that we are using a multiscale-based estimator, the resulting estimates are quite smooth and free of the artifacts that can appear in other multiscale models[4].

We will present further details of the multiscale modeling process, and discuss the statistical circumstances under which our selected class of multiscale models (4) are reasonable. We will also discuss the example hydrographic problem in greater detail, motivate an interest in the problem, and elucidate why it is that maps such as in Figure 5 are of such interest.

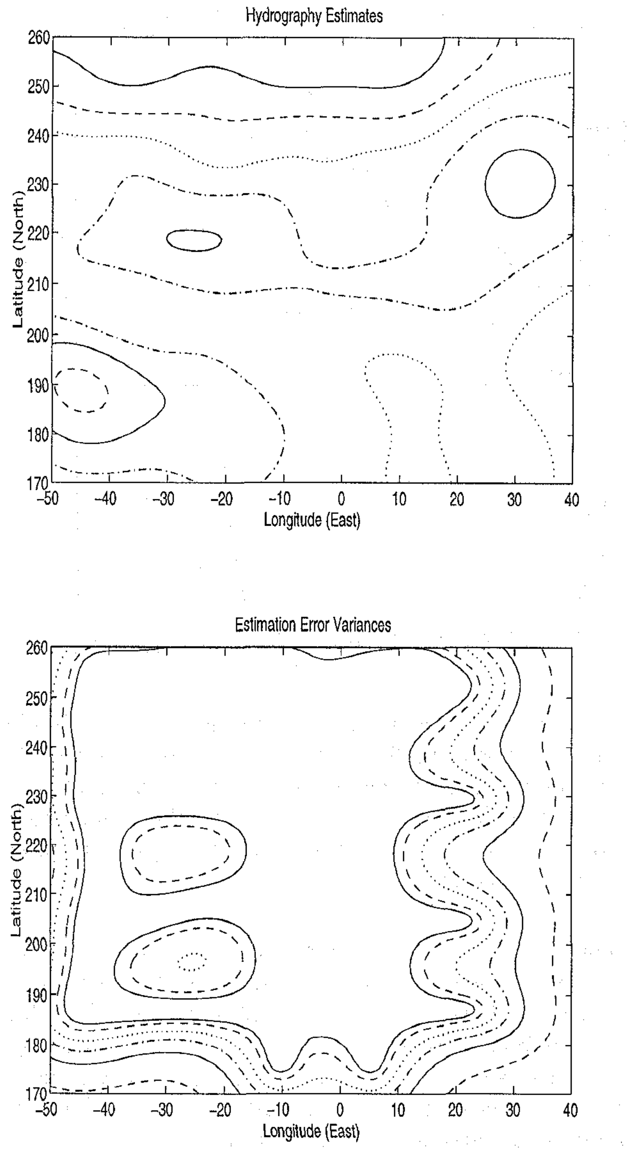

Figure 5: The dense estimate and error statistic maps produced by the multiscale model of Figure 4. Note that both of the maps are smooth and are not subject to discontinuities or artifacts along tree boundaries.

4. SOFTWARE

The multiscale estimation software used to produce the results of this paper are available for downloading via anonymous FTP from lids.mit.edu in the directory pub/ssg/code/Hydrography. The software is distributed *as is* (i.e., without support), however comments and bug reports are welcome and may be sent to the author at pwfiegut@cs.toronto.edu

5. REFERENCES

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