A PROBABILISTIC FRAMEWORK FOR IMAGE SEGMENTATION

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ABSTRACT

A new probabilistic image segmentation model based on hypothesis testing and Gibbs Random Fields is introduced. First, a probabilistic difference measure derived from a set of hypothesis tests is introduced. Next, a Gibbs/Markov Random Field model endowed with the new measure is then applied to the image segmentation problem to determine the segmented image directly through energy minimization. The Gibbs/Markov Random Fields approach permits us to construct a rigorous computational framework where local and regional constraints can be globally optimized. Results on grayscale and color images are encouraging.

1. INTRODUCTION

Humans can easily, even effortlessly distinguish between separate objects in an image scene. This has long been a key problem in computer vision, where a number of steps, from low-level to high-level vision, are needed to understand an image or some portion of it. A critical step, and the topic of this paper, is that of image segmentation.

Image segmentation is based on the idea that two separate objects with distinct appearance can be separated. In a greyscale image we rely on different brightness levels or textures; in a color image, the difference can be based on color differences; in the most general case of arbitrary multispectral images, the appropriate segmentation criteria will be problem-specific. We will focus on multispectral images.

The image segmentation process is dependent on two interactive components: 1) a measure of discontinuity and 2) a framework for grouping similar pixels and separating dissimilar ones. The idea of brightness discontinuity between objects in greyscale images is fundamental to the problem of image segmentation. It is used to devise schemes to segment images by separating homogenous brightness groupings from each other [3]. The main component in such an algorithm is the distance measure used to assess the similarity or discontinuity in brightness. Typically this distance measure is the Euclidean distance in 1-D; in other words, the simple difference [4].

However, the first question one needs to ask is whether the simple difference or Euclidean distance (for multispectral images) is appropriate for measuring brightness discontinuities. The Euclidean distance is used given its intuitive appeal and low computational complexity for multispectral images. However, there are many instances where this discontinuity measure fails: noisy images, specular reflections, intensity variations (such as those produced by shadows), etc. In color images, the vector angle is an intensity invariant distance measure [1]; however, the vector angle gives very "noisy" results for vectors with small magnitudes. A probabilistic framework would help to better define appropriate discontinuity measures given the assumptions of the particular problem at hand.

The second component, the pixel grouping algorithm, has taken on different forms. Image segmentation can be subdivided into three broad categories: point-based techniques [7], spatially or region based methods [6] and hybrids [8, 10]. Point-based methods are concerned with global pixel comparisons such as those done using clustering algorithms. The primary drawback of point-based techniques is their inability to take into account local variation to avoid the formation of many small extraneous regions, as well as the need of knowing how many cluster prototypes to select [8]. Region based methods are primarily based on spatial pixel proximity and similarity. Their main disadvantage is that a region could include two vastly different regions due to a small gradient between the two distinctly "colored" regions. Hybrid methods [8, 10] attempt to combine both the point-based and spatial paradigms. Methods developed based on both paradigms minimize the disadvantages from either group of methods. This paper will be concerned with the use of hybrid image segmentation algorithms based on Gibbs Random Fields.

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2. A PROBABILISTIC DISCONTINUITY MEASURE

A basic image segmentation algorithm is concerned with the question of whether two pixels are part of the same region/group or part of two different regions/groups. When some distance measure such as the Euclidean distance is used, the decision to include a pixel in one group or the other is essentially explicitly (e.g., region growing methods) or implicitly (e.g., number of regions in clustering methods) an experimentally set threshold. However, this type of discontinuity criterion does not take into account noise, and illumination effects.

One way to take into account noise is to assume a particular noise model and devise a discontinuity measure based on this assumption. Each pixel is assigned an integer label $0 \leq l < M$ to associate it with one of the M region prototypes. If we associate with label l a global region prototype $\{\vec{a}_l\}$ then each region has a well defined meaning. Consider the following model of a multispectral pixel, which is a standard assumption in noisy image restoration problems:

$$\vec{X}_{a_l} = \vec{a}_l + \vec{N}_{a_l} \quad (1)$$

where $\vec{X}_{a_l}$ is the pixel vector, $\vec{a}_l$ is the region prototype with some associated label l, and $\vec{N}_{a_l}$ is some prototype-dependent noise covariance R_{a_l}. In this paper, we will assume that the noise component for each prototype is Gaussian distributed.

Consider that there are two adjacent pixels $\vec{X}_{a_1}$ and $\vec{X}_{a_2}$ consisting of noise and unknown region prototypes $\vec{a}_1$ and $\vec{a}_2$ respectively. $\vec{a}_1$ and $\vec{a}_2$ may be equal or different. In practice, the pixels do not need to be adjacent and can be any two sets of vectors. However, given the Gibbs/Markov Random Field framework that will be used to perform the segmentation, the pixels will be considered adjacent. Consider that the image actually consists of two regions (i.e., $M = 2$) with prototypes $\vec{c}_l$. However, we do not know for which pixel the vector $\vec{a}_1$ corresponds to $\vec{c}_1$ and for which to $\vec{c}_2$ (and correspondingly for $\vec{a}_2$).

There are therefore four hypothesis tests that can be applied with the following respective null hypotheses:

Hypothesis Test 1: $\vec{a}_1 = \vec{a}_2 = \vec{c}_1$
Hypothesis Test 2: $\vec{a}_1 = \vec{a}_2 = \vec{c}_2$
Hypothesis Test 3: $\vec{a}_1 = \vec{c}_1, \vec{a}_2 = \vec{c}_2$
Hypothesis Test 4: $\vec{a}_1 = \vec{c}_2, \vec{a}_2 = \vec{c}_1$

Each hypothesis test implies probability densities for $\vec{X}_{a_1}$ and $\vec{X}_{a_2}$. Each of the hypothesis tests results in a probability density function with means set to $\vec{c}_1$ or $\vec{c}_2$ depending on the hypothesis being tested. For example, for Hypothesis Test 1 both $\vec{X}_{a_1}$ and $\vec{X}_{a_2}$ would be distributed as Gaussians as follows:

$$\begin{aligned} p(\vec{X}_{a_1}|HT1) &\sim \mathcal{N}(\vec{c}_1, R_{c_1}) \\ p(\vec{X}_{a_2}|HT1) &\sim \mathcal{N}(\vec{c}_1, R_{c_1}) \end{aligned} \quad (2)$$

Assuming pixels $\vec{X}_{a_1}$ and $\vec{X}_{a_2}$ are independent we have

$$p(\vec{X}_{a_1}\vec{X}_{a_2}|HT1) = p(\vec{X}_{a_1}|HT1)p(\vec{X}_{a_2}|HT1) \quad (3)$$

We would now end up with four joint probability densities $p(\vec{X}_{a_1}\vec{X}_{a_2}|HT1) \ldots p(\vec{X}_{a_1}\vec{X}_{a_2}|HT4)$. The highest joint probability for particular $\vec{X}_{a_1}$ and $\vec{X}_{a_2}$ would be chosen as the winner and would tell us which labels $\vec{X}_{a_1}$ and $\vec{X}_{a_2}$ should assume. The joint probability density can be rewritten as an Gibbs energy function by examining the variables in the exponent from the Gaussian densities in (2):

$$\begin{aligned} \Phi(\vec{X}_{a_1}, \vec{X}_{a_2}) = & (\vec{X}_{a_1} - \vec{c}_1)^T R_{c_1}^{-1}(\vec{X}_{a_1} - \vec{c}_1) + \\ & (\vec{X}_{a_2} - \vec{c}_1)^T R_{c_1}^{-1}(\vec{X}_{a_2} - \vec{c}_1) \end{aligned} \quad (4)$$

We call this the probabilistic discontinuity measure. In other words, we are introducing a modification to the standard Euclidean distance formulation by making the result dependent on R, the covariance matrix of the noise. We were thus able to rewrite the hypothesis test into an energy function. If we set $R = I$, then we obtain the Euclidean distance.

The theory behind this approach is well grounded in statistics and therefore there is nothing new from that point of view. However, this methodology is being applied to image segmentation for the first time (as far as we know) and as such presents a new way of looking at this important problem. The idea presented here is to consider any region homogenous in "color" or characteristics corrupted with some type of noise, as well as other effects such as specular reflections. In this paper, we only address the case of noise "corruption".

Furthermore, we make the assumption that we know the noise model (Gaussian) and estimate its parameters (vector means and covariance matrices). How this is achieved will be addressed in the next section.

Finally, for real world problems where typically $M > 2$, this method would quickly become impractical given the high number of hypothesis tests necessary. To avoid this problem, it would be necessary to reformulate the hypothesis tests into the following one:

$$\begin{aligned} H0 &: \vec{a}_1 = \vec{a}_2 \\ H1 &: \vec{a}_1 \neq \vec{a}_2 \end{aligned} \quad (5)$$

With this formulation, we can also keep equation (4).

3. MARKOV RANDOM FIELDS

The modeling problems in this paper are addressed from the computational viewpoint by using Markov random fields to model the image segmentation process. There are two primary concerns: how to define an objective function for the optimal solution of the image segmentation, and how to find its optimal solution. It is reasonable to define the desired solution in an optimization sense given the various uncertainties in the imaging process. In this case, the "exact" solution to our segmentation problem would be interpreted as the optimum solution to the optimization objective.

Some forms of contextual constraints are eventually necessary when trying to interpret visual information. Markov Random Fields (MRF) [2, 9] provide a natural way of modeling context dependencies between, for example, image pixels of correlated local features [5]. However, instead of using the Gaussian Markov Random Field to define a spatial texture from which segmentation can proceed as a separate hypothesis-testing procedure [5], our approach finds the segmented image directly as the result of energy minimization of some appropriately-defined Gibbs random field. Furthermore the regions are not distinguished on the basis of texture, rather on intensity differences (whether in one or more dimensions). The formulation of our Gibbs/Markov model will be similar to others used for segmentation [2, 5] except that we consider global/regional constraints in addition to local ones used in classic MRF research. Since our goal is the assertion of local constraints, rather than an accurate modeling of spatial textures [5], we shall only be concerned with first order random fields (i.e., a 4 pixel neighborhood).

This paper builds on previous work in color image segmentation using Markov Random Fields [1, 8] and extends it to an MRF framework based on hypothesis tests. The principles presented in this paper can be applied to any multispectral images as long as the probabilistic discontinuity measure is derived for the particular problem at hand.

As described above, the basic idea is to replace the difference calculations within the model with the probabilistic discontinuity measure. Therefore, energy cliques based on adjacent pixels would be calculated according to (4) rather than using Euclidean distance or vector angle [8]. Suppose we are given a multispectral image $\vec{X}$ on a pixel lattice $\mathcal{L} = \{i, j\}$. The energy model is shown as follows:

$$\begin{aligned} H = \sum_{i,j} \Phi(\vec{X}_{i,j}, \vec{X}_{i,j+1})\delta_{l(i,j),l(i,j+1)} + \\ \Phi(\vec{X}_{i,j}, \vec{X}_{i+1,j})\delta_{l(i,j),l(i+1,j)} + \\ \beta\left[(1 - \delta_{l(i,j),l(i,j+1)}) + (1 - \delta_{l(i,j),l(i+1,j)})\right] \end{aligned} \quad (6)$$

where β controls the relative constraints on the degree of region cohesion and fragmentation, while $\delta_{l(i,j),l(i,j+1)}$ is 1 when both labels are the same and 0 otherwise. The δ functions ensure that the labelling is consistent. If the labelling is inconsistent then the energy function will be high.

Given equation (3), we can represent (6) as a joint probability density over $\vec{X}_{i,j}$ and all of its neighbors by multiplying all four (i.e., for a first order Markov model) joint probabilities or instances of (3). This is possible since each neighborhood comparison is independent of each other.

The region prototypes $\{\vec{c}_l\}$ can adapt via continuous Gibbs sampling [8]; in other words, the sampling and annealing takes place not only over label indices $\{l(i,j)\}$, but also over the continuous valued region prototypes $\{\vec{c}_l\}$. However, for this paper, we approximate this by computing the vector means instead. When labels are sampled, we use the probabilistic discontinuity measure to compute the distance between the pixel $\vec{X}_{i,j}$ and the corresponding region prototype $\{\vec{c}_l\}$. The distance is also computed between pixels $\vec{X}_{i+1,j}$ or $\vec{X}_{i,j+1}$ and their corresponding *fixed* region prototypes. This is possible since hypothesis test (5) does not ask for sampling over those prototypes, just the one being compared to the central pixel $\vec{X}_{i,j}$.

The general color image segmentation algorithm is now described:

- All pixel labels are randomly initialized
- The vector mean $\{\vec{c}_l\}$ and noise covariances are estimated for each cluster
- Repeat for several iterations:
  - At each pixel in the image:
    * Minimize the energy in model (6) by performing the hypothesis test (5) with respect to every region prototype (effectively sampling the labels)
    * Update the pixel's label based on the Gibbs sampler result
  - Use continuous Gibbs sampling to adjust the region prototypes $\{\vec{c}_l\}$ and compute the covariance matrix
  - Lower the temperature T

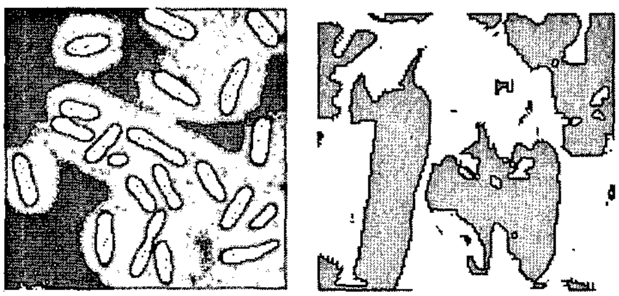
If the temperature reduction occurs slowly enough, this annealing process converges in probability to the global minimum [2].

4. RESULTS

Results were obtained on grayscale and color images shown in Figure 1. Image segmentation results for the grayscale image using 3 clusters and for the color image using 2 clusters are shown in Figure 2. The grayscale segmentation shows three distinct clusters at three intensity levels separating the features of interest which are the bacteria. It is apparent that the MRF might have not converged fully given the small dots present here and there. In the color segmentation, the color separation between the red and green papers is quite successful except for areas with illumination effects such as highlights.

(a) (b)

Fig. 1. (a) Grayscale image, (b) color image.



(a) (b)

Fig. 2. Segmentation results: (a) grayscale, (b) color.

5. CONCLUSIONS

A probabilistic framework for adaptive multispectral image segmentation using a probabilistic discontinuity measure and a Gibbs/Markov Random Field has been presented. The method presents several advantages: use of a discontinuity measure derived from first principles, adaptability of global constraints (region prototypes) to the data, sampling over both region labels and region prototypes using the Gibbs sampler (both discrete and continuous), optimization of local contextual constraints (taking into account local features) with a global energy function (making sure that regions are optimally segmented with respect to each other). Hybrid methods have been introduced recently including region computation. However, none of these methods provides a framework as flexible as MRFs which not only adaptively and globally optimizes local constraints, but can also easily integrate texture handling and edge detection (line processes) within the segmentation process. Finally, the introduction of a probabilistic discontinuity measure and encouraging results allows us to build a principled image segmentation theory.

6. REFERENCES

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