Adaptive nonlinear image denoising and restoration using a cooperative Bayesian estimation approach

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Abstract

A novel nonlinear cooperative approach to image denoising and restoration is presented. Samples from the image field with similar characteristics are first grouped into clusters by first performing image decomposition based on the Mumford-Shah model using a total variational framework and performing fuzzy c-means clustering within each image partition. Samples within each cluster are then aggregated using an cooperative Bayesian estimation method based on information from all the samples to provide a nonlinear estimate of the original image. The proposed method exploits information redundancy within each cluster to denoise and restore the original image. Furthermore, the proposed cooperative Bayesian estimation method is capable of suppressing noise and reducing image degradation while preserving image detail by utilizing intra-cluster statistics. The experimental results using different types of images demonstrate that the proposed algorithm provides state-of-the-art image denoising performance in terms of both peak signal-to-noise ratio (PSNR) and subjective visual quality.

1 Introduction

Image denoising is the process of estimating the original image from an image that has been contaminated by noise degradation. Image denoising is one of the fundamental challenges in the field of image processing and is important in a wide variety of applications such as object recognition, video tracking, photo enhancement and restoration, and image segmentation. Despite the large number of algorithms proposed in the area, the problem of noise suppression remains an open challenge in situations characterized by additional image degradation and low signal-to-noise ratios.

Many image denoising methods have been introduced and can generally be categorized as either transform domain methods or spatial domain methods. Transform domain methods first transform an image from the spatial domain into a different domain (e.g., frequency domain, wavelet domain) and suppress noise in the transform domain. Transform domain denoising methods include Wiener filtering [12], Gaussian scale mixture (GSM) denoising [10], wavelet shrinkage [7], collaborative Wiener filtering [3], and shape-adaptive discrete cosine transform (SA-DCT) denoising [5].

Spatial domain methods suppress noise directly in the spatial domain. These can be divided into two main groups: i) local spatial domain methods, and ii) global spatial domain methods. Local spatial domain methods take advantage of image redundancy within small neighborhoods to suppress noise. Such methods include Gaussian filtering, bilateral filtering [11, 4, 13], anisotropic filtering [6], and trilateral filtering [15]. The main advantage of local methods is that they are very efficient from a computational perspective. However, such methods perform poorly for images with low signal-to-noise ratios since local information does not provide enough information for proper noise suppression. Global spatial domain method takes advantage of image redundancy within the entire image and thus are better suited for situations characterized by low signal-to-noise ratios. These methods include non-local means [8, 2, 9], and anisotropic non-local means [14]. One common drawback associated with most spatial domain methods is that they do not account for additional image degradation which can make it difficult to perform proper denoising.

In this paper, we propose a novel approach to image denoising using a cooperative estimation scheme. The proposed method takes advantage of global information redundancy across similar image samples through the use of a novel cooperative Bayesian estimation scheme and is highly robust to images characterized by low signal-to-noise ratios and additional image degradation. Furthermore, the proposed method provides effective noise suppression while preserving image detail. This paper is organized as follows. The proposed method is presented in Section 2. Experimental results are pre-

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sented and discussed in Section 3. Finally, conclusions are drawn in Section 4.

2 Proposed Method

The proposed algorithm takes a cooperative approach to the problem of image denoising and restoration and can be described as follows. First, samples within an image field are clustered based on their similarity by first performing image decomposition based on the Mumford-Shah model using a total variational framework and performing fuzzy c-means clustering within each image partition. Second, a novel cooperative Bayesian estimation scheme is used to aggregate the samples adaptively within each cluster to produce a nonlinear estimate of the original image.

2.1 Background

Let f be a 2-D digital image, which is subsequently degraded by degradation function H and contaminated by additive noise n during the image acquisition process. The degraded image g acquired by the image acquisition system can be modelled as,

$$g(\underline{x}) = \mathrm{H}f(\underline{x}) + n(\underline{x}), \tag{1}$$

where $\underline{x} = (x, y)$. The goal of image denoising and restoration is to estimate f given g. Supposing that $\mathrm{H} = 1$, local spatial domain denoising methods estimate f as the weighted sum over a local spatial neighborhood ψ in g,

$$\hat{f}(\underline{x}) = \sum_{\forall \underline{x} \in \psi} w(\underline{x}) g(\underline{x}), \tag{2}$$

where w represents the weighting function as defined by the spatial filter. The above formulation for local spatial domain denoising is extended by global spatial domain methods to take into account the entire image g. Therefore, the estimate of $\hat{f}$ using the global spatial domain denoising approach be expressed as,

$$\hat{f}(\underline{x}) = \sum_{\forall \underline{x} \in g} w(\underline{x}) g(\underline{x}). \tag{3}$$

There are two main problems with existing global spatial domain methods. First, it is computationally impractical to estimate $\hat{f}$ based on Eq. (3), as it requires recomputing the weighting function for all pixels, which in itself requires evaluating the similarity of every pixel. As such, practical implementations of global spatial domain denoising are highly restrained to restricted search regions. Second, existing global spatial domain methods do not account for image degradation $\mathrm{H} \neq 1$, which is problematic as it is often difficult to decouple image degradation and image noise in practical situations. Finally, such global spatial domain methods do not explicitly account for local image statistics, which is important for image detail preservation. We aim to address all of these issues through the use of a novel cooperative estimation scheme.

2.2 Sample Clustering

The first step of the proposed method is to cluster image samples from g based on their similarity. There are two main reasons for performing this clustering process. First, it allows cooperative estimation to be performed between samples within smaller clusters, thus significantly reducing the computational complexity of the denoising and restoration process while still exploiting the global information redundancy within the image. Second, by clustering image samples that are similar, it reduces the effect of irrelevant image samples on the image denoising and restoration process. Unfortunately, it is difficult to perform accurate sample clustering under situations characterized by image degradation and noise. To address this issue, we introduce a modified fuzzy c-means clustering scheme based on Mumford-Shah image decomposition using a total variational framework.

The proposed sample clustering algorithm can be described as follows. The degraded image g is first decomposed into piecewise-smooth regions u and textured regions v by utilizing the two-phase piecewise smooth approximation of the Mumford-Shah model using a total variational framework [1],

$$min\left\{E(k,\vec{C}) = \beta \iint_{\Omega} \|k-g\|^2 dA + \alpha \iint_{\Omega\vec{C}} \|\nabla k\|^2 dA + \gamma \oint_{\vec{C}} ds\right\}, \tag{4}$$

where $k = u + v$, Ω represents the image domain, $\vec{C}$ is a close subset of Ω, and α, β, γ are arbitrary constants. For a region u_i (or v_i), fuzzy c-means clustering is performed to obtain m clusters $L_1, L_2, ..., L_m$ based on the fuzzy weighting function w_t, which is defined as a product of a intensity weighting function w_i and a spatial weighting function w_s and can be expressed as,

$$w_t(p,q) = w_i(p,q) w_s(p,q), \tag{5}$$

and,

$$w_i(p,q) = e^{\frac{-[(g(p)-g(q))]^2}{2\sigma_i^2}} \tag{6}$$

$$w_s(p,q) = e^{\frac{-\|p-q\|^2}{2\sigma_s^2}}, \tag{7}$$

where σ_i^2 and σ_s^2 are the penalty terms for intensity and spatial dissimilarity, p and q are the prototype and candidate sample respectively. The pixel samples within each cluster can then be used to estimate the original image f in a cooperative manner.

2.3 Cooperative Bayesian Estimation

Given a cluster L_i of k image samples $g_{[L_i]} = \{g(\underline{x}_1), g(\underline{x}_2), ..., g(\underline{x}_k)\}$, a total of k image estimates are produced $\{\hat{f}(\underline{x}_1), \hat{f}(\underline{x}_2), ..., \hat{f}(\underline{x}_k)\}$ using a novel cooperative Bayesian estimation scheme. The proposed cooperative estimation scheme extends the concepts of global spatial domain denoising in several important ways. First, unlike the original global spatial domain denoising methods, the proposed cooperative estimation scheme also accounts for image degradation in addition to image noise. Second, the proposed estimation scheme takes advantage of global information redundancy in a cooperative manner as well as accounting for intra-cluster spatial locality.

The proposed cooperative Bayesian estimation scheme can be defined in the following manner. Given k image samples $g_{[L_i]} = \{g(\underline{x}_1), g(\underline{x}_2), ..., g(\underline{x}_k)\}$, The estimate of $f(\underline{x}_i)$ can be represented as a linear combination of $\{g(\underline{x}_1), g(\underline{x}_2), ..., g(\underline{x}_k)\}$,

$$\hat{f}(\underline{x}_i) = \sum_{i=1}^{k} a_i g(\underline{x}_i) + \mu_g. \tag{8}$$

Now consider $g_{[L_i]}$ and f as two random vectors, H is the degradation matrix, and $\Re_n$ is noise variance. The image estimate $\hat{f}$ can be written to take the form of,

$$\hat{f} = \mathbf{A} g_{[L_i]} + b. \tag{9}$$

Based on Eq. (9), the underlying goal of the proposed method is to determine the values of A and b using unbiasedness and orthogonality conditions, which states the estimator is unbiased and the estimation error is perpendicular to any linear combination of f,

$$E\left[(f - \hat{f})^2\right] = 0, \tag{10}$$

and

$$E\left[(f - \hat{f})(\kappa g_{[L_i]} + \gamma)\right] = 0. \tag{11}$$

By enforcing the conditions expressed in Eq. (10) and Eq. (11), the estimate of f can be defined in the Bayesian sense as,

$$\hat{f} = \mu_f + (\mathrm{H}^T \Re_n^{-1} \mathrm{H} + \Re_{ff}^{-1})^{-1} \mathrm{H}^T \Re_n^{-1} \left[g_{[L_i]} - \mathrm{H}\mu_f\right]. \tag{12}$$

This is a nice close-form solution for the proposed estimation method, but in reality the mean and variance of f and g are not known in advance, those making the estimation problem difficult to solve in an ideal sense. Therefore, an approximate solution is desired in this situation.

In the proposed algorithm, the approximate solution is determined based on the intra-cluster image statistics of L_i. Therefore, the Bayesian estimator expressed in Eq. (12) can be re-formulated based on the intra-cluster image stastics as,

$$\hat{f} = \widehat{\mu}_g + (\frac{h^2}{\widehat{\sigma}_n^2} + \frac{1}{\widehat{\sigma}_g^2})^{-1} \frac{h}{\widehat{\sigma}_n^2} (g_{[L_i]} - h\widehat{\mu}_g), \tag{13}$$

where σ_n^2 is the estimated variance of the noise, $\widehat{\mu}_g$ and $\widehat{\sigma}_g^2$ are the cooperative mean and variance at sample $\underline{x}_c$, as defined by,

$$\widehat{\mu}_g = \frac{\sum_{i=1}^{k} g\left(\underline{x}_i\right) \exp\left(\frac{\|g(\psi_i) - g(\psi_c)\|_2}{\vartheta}\right)}{\sum_{i=1}^{k} \exp\left(\frac{\|g(\psi_i) - g(\psi_c)\|_2}{\vartheta}\right)} \tag{14}$$

$$\widehat{\sigma}_g^2 = \frac{1}{k-1} \sum_{i=1}^{k} \left(g\left(\underline{x}_i\right) - \mu_c\right)^2 \tag{15}$$

where ψ_i and ψ_c are local neighborhoods at $\underline{x}_i$ and $\underline{x}_c$ respectively, $\|.\|_2$ is the L^2-norm, and ϑ is the decay coefficient. One issue with the above formulation is that it is difficult to know the actual degradation model. To address this issue, the proposed approach imposes an isotropic penalty (α = 0) for smooth regions and non-isotropic penalty ($\alpha < 0$) for textured regions. The resulting degradation term h can be expressed as,

$$h = \begin{bmatrix} 0 & -\frac{1}{4} & 0 \\ -\frac{1}{4} & 1+\alpha & -\frac{1}{4} \\ 0 & -\frac{1}{4} & 0 \end{bmatrix}, \tag{16}$$

where α is the penalty term. It can be observed the the aforementioned cooperative estimation method adapts based on the characteristics of the other images samples within the cluster. As such, the different samples in the cluster cooperate to suppress noise and image degradation while preserving image detail.

3 Experimental Results

The proposed adaptive cooperative denoising and restoration method was tested on a set of five images with different image content characteristics. For each image, two different test scenarios were evaluated: i) Gaussian white noise with standard deviation of 51, and

ii) motion blur and Gaussian white noise with standard deviation of 13. To evaluate the effectiveness of the proposed method in a quantitative manner, the peak signal-to-noise ratio (PSNR) of the restored image was computed for the proposed method along with state-of-the-art denoising methods such as bilateral filtering (BF) [11], Gaussian scale mixture (GSM) denoising [10], and non-local means (NLM) denoising [2].

A summary of the PSNR results for the test scenario with noise is shown in Table 1. It can be observed that the proposed method achieves noticeable PSNR gains compared the BF and NLM denoising methods for all images. When compared to the GSM denoising method, the proposed method produced comparable results for the Lena, Boat, and Barbara test cases and noticeable PSNR gains for the House test case. The restored images are shown in Figure 1. It can be observed that the proposed method produced restored images with noticeably better perceptual quality when compared to existing methods. Fine image detail is better preserved by the proposed method compared to the other tested methods, while still achieving high noise suppression and restoration performance.

A summary of the PSNR results for the test scenario with noise and image degradation is shown in Table 2. It can be observed that the proposed method achieves noticeable PSNR gains compared all tested methods for all images. The restored images are shown in Figure 2. It can be observed that, like the first test scenario. the proposed method produced restored images with noticeably better perceptual quality when compared to existing methods. Furthermore, it can be noticed that the proposed method significantly reduced the motion blur degradation found in the degraded image. As a result, the restored images produced by the proposed method are significantly sharper and more detailed than that produced using the other tested methods. This demonstrate the effectiveness of the proposed method for suppressing noise and degradation even in situations characterized by low signal-to-noise ratios.

4 Conclusions

In this paper, a novel denoising and restoration method based on cooperative estimation was introduced. The proposed method utilizes information redundancy across the image in an efficient manner through sample clustering and a nonlinear cooperative Bayesian estimation scheme. The proposed method adapts based on intra-cluster image characteristics to suppress noise and degradation while preserving image detail. Experimental results demonstrate that state-of-the image denoising and restoration performance can be achieved to produce restored image with high perceptual quality. Future work involves investigating alternative clustering methods for improving inter-cluster similarity for better noise suppression and restoration performance.

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Table 1. PSNR for Test Images (noise)

Test	PSNR (dB)			
	GSM [10]	BF [11]	NLM [2]	Proposed
Lena (14.46)	27.35	24.25	25.6	27.34
Boat (15.02)	24.62	23.58	23.07	24.77
House (14.45)	27.7	24.18	24.60	27.50
Barbara (14.61)	23.59	22.33	22.53	23.82

Table 2. PSNR for Test Images (noise and image degradation)

Test	PSNR (dB)			
	GSM [10]	BF [11]	NLM [2]	Proposed
Lena (16.83)	17.47	17.48	18.02	24.41
Boat (17.11)	17.80	17.83	18.38	24.43
House (18.41)	19.35	19.33	20.10	26.83
Barbara (15.85)	16.35	16.37	16.83	21.63

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(a) Lena

(b) Boat

(c) House

(d) Barbara

Figure 1. Denoising using different denoising methods without image degradation (from left to right): a) Noisy image (σ=51), b) BF c) GSM, d) NLM, e) Proposed

(a) Lena

(b) Boat

(c) House

(d) Barbara

Figure 2. Denoising using different denoising methods with image degradation (from left to right): a) Noisy and degraded image (σ=13), b) BF c) GSM, d) NLM, e) Proposed