ADAPTIVE WIENER FILTERING OF NOISY IMAGES AND IMAGE SEQUENCES

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ABSTRACT

In this work, we consider the adaptive Wiener filtering of noisy images and image sequences. We begin by using an adaptive weighted averaging (AWA) approach to estimate the second-order statistics required by the Wiener filter. Experimentally, the resulting Wiener filter is improved by about 1dB in the sense of peak-to-peak SNR (PSNR). Also, the subjective improvement is significant in that the annoying boundary noise, common with the traditional Wiener filter, has been greatly suppressed.

The second, and more substantial, part of this paper extends the AWA concept to the wavelet domain. The proposed AWA wavelet Wiener filter is superior to the traditional wavelet Wiener filter by about 0.5dB (PSNR). Furthermore, an interesting method to effectively combine the denoising results from both wavelet and spatial domains is shown and discussed. Our experimental results outperform or are comparable to state-of-art methods.

1. INTRODUCTION

Images and image sequences are frequently corrupted by noise in the acquisition and transmission phases. The goal of denoising is to remove the noise, both for aesthetic and compression reasons, while retaining as much as possible the important signal features. Very commonly, this is achieved by approaches such as Wiener filtering [1, 2], which is the optimal estimator (in the sense of mean squared error (MSE)) for stationary Gaussian process.

Since natural images typically consist of smooth areas, textures, and edges, they are clearly not *globally* stationary. Similarly, nonstationarity in video may further be caused by inter-frame motion. However, image and video can be reasonably treated as being *locally* stationary, as shown by Kuan [1] and Lee [2] for images, and similar arguments can be made for motion-compensated video.

These insights have motivated the design of adaptive Wiener filters, called local linear minimum mean square error (LLMMSE) filters. The LLMMSE filter proposed by Lee [2] (the so-called Lee filter), extensively used for video denoising, is successful in the sense that it effectively removes noise while preserving important image features (eg., edges). However the Lee filter suffers from annoying noise around edges, due to the assumption that all samples within a local window are from the same ensemble. This assumption is invalidated if there is a sharp edge within the window, for example; in particular, the sample variance near an edge will be biased large because samples from two different ensembles are combined, and similarly the sample mean will tend to smear. The main problem, then, is how to effectively estimate local statistics.

More recently there has been considerable attention paid to wavelet-based denoising because of its effectiveness and simplicity. Both wavelet shrinkage [3, 4] and wavelet Wiener [5, 6] methods have shown to be very effective in signal and image denoising, although the latter Wiener approach is the one of interest in our context. It is well established that the wavelet transform is an effective decorrelator, and thus a reasonable approximation to the Karhuen-Loeve basis. Consequently a local wavelet Wiener filter should be more effective than its spatial counterpart, however the nonstationary local second order statistics must still be estimated.

In this paper we formally develop adaptively weighted averaging (AWA), proposed by Ozkan *et al* [7], however our work differs from [7] in that we use AWA to estimate local statistics instead of using it *directly* for denoising. A final section illustrates an effective way to combine our spatial and wavelet-based AWA filtering results. Experimental results confirm a significant improvement in image denoising.

2. LOCAL ADAPTIVE WIENER FILTERING

Consider the filtering of images corrupted by signal-independent zero-mean white Gaussian noise. The problem can be modeled as

$$y(i,j) = x(i,j) + n(i,j) \qquad (1)$$

where $y(i,j)$ is the noisy measurement, $x(i,j)$ is the noise-free image and $n(i,j)$ is additive Gaussian noise. The goal

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is to remove noise, or "denoise" $y(i,j)$, and to obtain a linear estimate $\hat{x}(i,j)$ of $x(i,j)$ which minimizes the mean squared error (MSE),

$$MSE(\hat{\mathbf{x}}) = \frac{1}{N}\sum_{i,j=1}^{N}(\hat{x}(i,j)-x(i,j))^2 \qquad (2)$$

where N is the number of elements in $x(i,j)$.

When $x(i,j)$ and $n(i,j)$ are stationary Gaussian processes the Wiener filter is the optimal filter [1]. Specifically, when $x(i,j)$ is also a white Gaussian process the Wiener filter has a very simple scalar form:

$$\hat{x}(i,j) = \frac{\sigma_x^2(i,j)}{\sigma_x^2(i,j)+\sigma_n^2(i,j)}[y(i,j)-\mu_x(i,j)] + \mu_x(i,j) \qquad (3)$$

where σ^2, μ are the signal variances and means, respectively, and where we will normally assume the mean of the noise to be zero. The effectiveness of the simple form Wiener filter (3) was documented in [1, 2]. In particular, Kuan proposed a nonstationary mean and nonstationary variance (NMNV) image model; conditioned on this model, for natural images the residual process can be well treated as white Gaussian processes.

To use (3) we need to determine $\mu_x(i,j)$, $\sigma_x^2(i,j)$ and $\sigma_n^2(i,j)$. We will assume that the noise mean and variance are known (for the well-established problem of noise-variance estimation readers are referred to [3, 4] and references therein). Instead, we focus on the local estimation of $\mu_x(i,j)$ and $\sigma_x^2(i,j)$. Normally [2] the local mean and local variance are calculated over a uniform moving average window of size $(2r+1)\times(2r+1)$:

$$\hat{\mu}_x(i,j) = \frac{1}{(2r+1)^2}\sum_{p=i-r}^{i+r}\sum_{q=j-r}^{j+r} y(p,q) \qquad (4)$$

$$\hat{\sigma}_x^2(i,j) = \frac{1}{(2r+1)^2}\sum_{p=i-r}^{i+r}\sum_{q=j-r}^{j+r}(y(p,q)-\hat{\mu}_x(i,j))^2 - \sigma_n^2 \qquad (5)$$

As discussed in the Introduction, (4) and (5) tend to blur the mean and increase the variance near edges. Thus, the resulting denoised image is poor and looks noisy (Fig. 1(c)).

Kuan *et al.* [1] proposed using a weighted form of (5) to estimate $\sigma_x^2(i,j)$ while still using (4) as the estimate of $\mu_x(i,j)$:

$$\hat{\sigma}_x^2(i,j) = \sum_{p=i-r}^{i+r}\sum_{q=j-r}^{j+r} w(i,j,p,q)(y(p,q)-\hat{\mu}_x(i,j))^2 \qquad (6)$$

To determine the nonstationary weights $w(i,j,p,q)$ Kuan suggested using a monotonically decreasing function (e.g., Gaussian) to put more confidence on the center variance estimates, however the idea was not developed formally.

Rather than a deterministic Gaussian weight, for an image which may contain abrupt edges and other changes in behaviour, it is far more appropriate to consider an adaptive approach to selecting $w()$. For example, the pixels used to compute the local variance σ^2 of some point (i,j) should be biased in favour of those pixels having values similar to $y(i,j)$:

$$w(i,j,p,q) = \frac{K(i,j)}{1+a(max[\epsilon^2,(y(i,j)-y(p,q))^2])} \qquad (7)$$

where we assert that $w(i,j,i,j)=0$, and $K(i,j)$ is a normalization constant, given by

$$K(i,j) = \left\{\sum_{p,q}\frac{1}{1+a(max[\epsilon^2,(y(i,j)-y(p,q))^2])}\right\}^{-1} \qquad (8)$$

The quantities $a>0$ and $\epsilon = 2.5\sigma_n$ are the parameters of the weight function (see [7] for the determination of these parameters). We choose a such that $a\epsilon^2 \gg 1$ to exclude outliers from the weight function $w()$. Given $w()$ we estimate both the local mean and the local variance adaptively as

$$\hat{\mu}_x(i,j) = \sum_{p=i-r}^{i+r}\sum_{q=j-r}^{j+r} w(i,j,p,q)y(p,q) \qquad (9)$$

$$\hat{\sigma}_x^2(i,j) = \sum_{p=i-r}^{i+r}\sum_{q=j-r}^{j+r} w(i,j,p,q)(y(p,q)-\hat{\mu}_x(i,j))^2 \qquad (10)$$

In summary, our AWA-based parameter estimation aims, as much as possible, to use samples belonging to one consistent class in estimating μ_x and σ_x^2, which should lead to improved performance near edges. Our method is different from Kuan's [1] in three respects:

1. In [1] only the local variance is estimated in a weighted form. In comparison, we apply AWA to estimate both local mean and variance, which should reduce mean blur effects near edges.

2. Kuan put more confidence on the center estimates, whereas we set the center weights to zero, which we have experimentally found to better suppress singularities, especially in smooth regions.

3. Kuan's weights are deterministic and not adaptive to image features, whereas we are adapting to edge and other abrupt features.

3. LOCAL ADAPTIVE WAVELET WIENER

Recently, wavelet-based denoising has attracted extensive attention because of its effectiveness and simplicity. The most common wavelet denoising methods can be classified into two groups: shrinkage [3, 4] and wavelet Wiener [5, 6]. The intuition behind wavelet shrinkage the wavelet transform's effectiveness at energy compaction allows small coefficients to be interpreted as noise, and large coefficients as important signal features.

The wavelet Wiener method is based on the observation that because a natural image can be well modeled in the spatial domain as a NMNV Gaussian random process, from which it follows that the wavelet coefficients can are similarly NMNV Gaussian. By properly estimating local means and variances wavelet Wiener has comparable denoising performance to wavelet shrinkage [4, 5].

Based on the success of AWA-based spatial Wiener filtering, we wish to further develop these ideas in the wavelet domain. However several points should be emphasized:

1. The mean values of all subbands above the lowest frequency are very small, and can reasonably be assumed to be zero. The only problem detected with this assumption is that the denoised images suffer from more ripple-like artifacts around edges. Conversely, using an AWA-estimated local mean yields much better edges but leads to structured artifacts in smooth regions. In the presented experiments we use a zero mean assumption, therefore only the local variance is estimated.

2. Although the wavelet transform is an effective decorrelator, there do remain structured correlations among the wavelet coefficients [6]. For example, the horizontal high frequency subband has much stronger correlation in the horizontal than in the vertical direction. Therefore the shape of the adaptive window really should be modulated based on some prior understanding of wavelet statistics; this more advanced approach is left as a future direction, and is not the focus of this paper.

4. COMBINED DENOISING

Although there have been are many attempts [8] to combine spatial and temporal denoising results in image sequence denoising, we are not aware of any other work in the literature that tries to combine spatial and wavelet denoising results. Because the remaining noise has quite different structures in the spatial and wavelet domains (we have dot-like remaining noise in the spatial domain and ripple-like remaining noise in the wavelet domain), we hope to suppress them further by taking advantage of this difference. Theoretically, if the two error images are uncorrelated we can get a gain of about 3dB in PSNR. Experimentally, the two error images are correlated, of course, as the error is mostly concentrated around edges, however the correlation coefficient relatively low (about 0.5), so experimental results show an improvement in PSNR of about 0.5dB. The subjective improvement is also considerable. Our proposed combination equation is shown below:

$$\hat{x}_{comb} = 0.6\hat{x}_{AWA-wavelet} + 0.4\hat{x}_{AWA-spatial} \quad (11)$$

where $\hat{x}_{AWA-wavelet}$ and $\hat{x}_{AWA-spatial}$ are the denoised results in the wavelet and spatial domain. The weights (0.6, 0.4) in are chosen to emphasize the observation that the MSE in the wavelet domain tends to be smaller than that in the spatial domain. Theoretically, optimal combination weights should be the function of the correlations and variances in the estimation errors.

5. RESULTS AND DISCUSSION

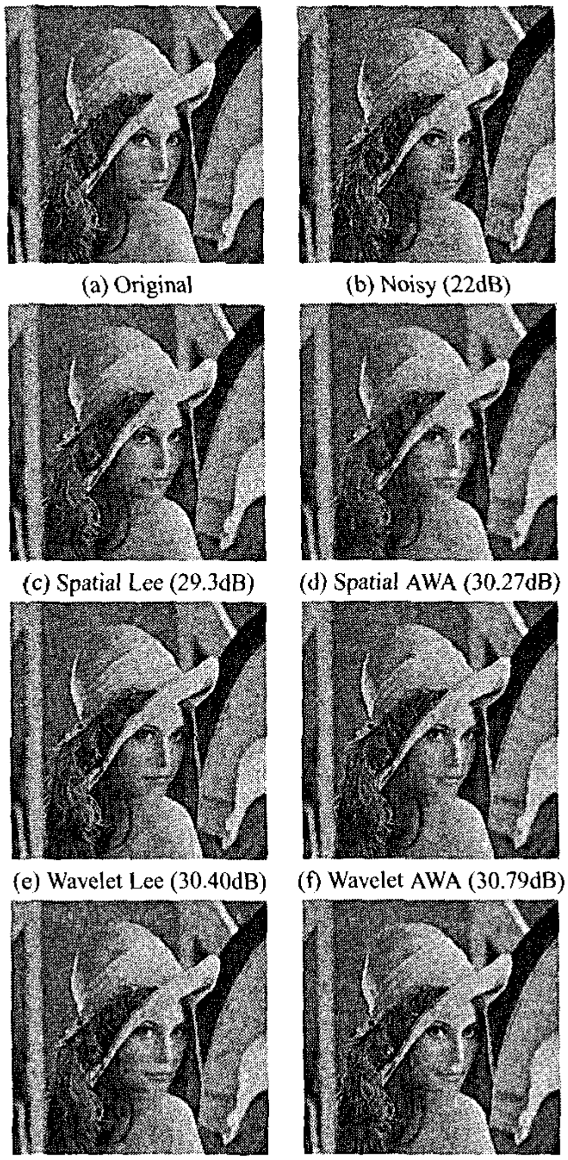
We first apply the developed AWA method (in both the spatial and wavelet domain) to noisy image Lena. The denoised results are shown in Fig.1.

The main observations of this experiment are

1. In the sense of PSNR the spatial AWA filter outperforms the spatial Lee filter by about 1dB-1.5dB. However, subjectively the spatial AWA filter tends to oversmooth edges. It seems to us that this problem can be well handled by adapting AWA method to the activity of different regions. Specifically, at smooth areas the center sample in the moving window should be neglected to suppress subjectively annoying singularities, whereas in rough areas the center sample should be properly used.

2. In the sense of PSNR the wavelet-based denoising outperforms the spatially denoising by about 0.5dB. This is mainly due to the energy compaction ability of wavelet transforms. Subjectively, the wavelet-based denoising methods preserve more details. The main problem with wavelet-based denoising methods are the ripple-like artifacts around edges. The wavelet-based AWA approach can effectively suppress the artifacts.

3. Experimentally we find that properly combining the wavelet-based and spatially denoising results can further improve PSNR by about 0.5dB. Subjective performance of the combination result is also considerably improved.



(a) Original (b) Noisy (22dB)

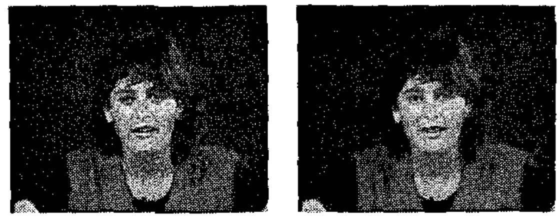
(c) Spatial Lee (29.3dB) (d) Spatial AWA (30.27dB)

(e) Wavelet Lee (30.40dB) (f) Wavelet AWA (30.79dB)

(g) Combined (31.28dB) (h) Bayeshrink (30.5dB)

Fig. 1. Comparing the results of various methods. PSNRs are shown in the brackets.

In the second experiment we apply AWA denoising methods to the image sequence *Missa*. To filter image sequences we use 3-D AWA method which is an extension of the proposed 2-D AWA method. We use simple block matching for motion estimation. The block size is 16×16. We observe that the 3-D AWA method can well adapt to the error of motion estimation and sudden scene change.



(a) (b)

Fig. 2. Denoising result for the third frame of the *Missa* sequence. (a) Noisy observation (PSNR=26dB), (b) Combined filtering (PSNR=36.5dB)

The average improvement of PSNR is above 10dB. Figure 2 shows the denoising result of the third frame of *Missa*.

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