Accurate Boundary Localization using Dynamic Programming on Snakes

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Abstract:

The extraction of contours using deformable models, such as snakes, is a problem of great interest in computer vision, particularly in areas of medical imaging and tracking. Snakes have been widely studied, and many methods are available. In most cases, the snake converges towards the optimal contour by minimizing a sum of internal (prior) and external (image measurement) energy terms. This approach is elegant, but frequently mis-converges in the presence of noise or complex contours. To address these limitations, this poster address a novel discrete snake, which treats the two energy terms separately. Essentially, the proposed method is a deterministic iterative statistical data fusion approach, in which the visual boundaries of the object are extracted, ignoring any prior, employing a Hidden Markov Model (HMM) and Viterbi search, and then applying importance sampling to the boundary points, on which the shape prior is asserted.

Introduction

Locating the exact boundaries of objects has many applications in object tracking, content based image and video retrieval systems, robotics and biomedical engineering. Energy minimizing splines, such as deformable snakes or active contours, are the key approaches in the computer vision literature for such boundary extraction problems.

A deformable model or snake is an energy minimizing spline which is mathematically represented as follows.

$$\mathbf{v}(s) = [x(s), y(s)], s \in [0, L] \tag{1}$$

$$E_{total} = \int_0^L [E_{\text{int}}(\mathbf{v}) + \gamma E_{ext}(\mathbf{v})] ds \tag{2}$$

$$\mathrm{E}_{\text{int}}(\mathbf{v}) = \underbrace{\alpha \left|\frac{\partial \mathbf{v}}{\mathrm{d}s}\right|^2}_{elasticity} + \underbrace{\beta \left|\frac{\partial^2 \mathbf{v}}{\mathrm{d}s^2}\right|^2}_{rigidity} \tag{3}$$

By definition, the optimum boundary is the one which minimizes total energy of the snake, whose closed form solution is not trivial (essentially impossible because of the clutter and complexity of the external energy term). Therefore, in the absence of closed-form solutions, iterative dynamic curve evolution methods are adopted to minimize total energy numerically. The problems that are not addressed in current literature are given below.

1.If initial snake is far from the object boundary, the external force is not able to attract the snake towards true boundary.

2.Final snake do not converge towards high curvature concave and convex boundaries

3.standard snake algorithms do not guarantee convergence and tend to be very sensitive to noise and false weak edges

4.A key problem, not currently addressed in the literature, is that parameters α, β, and γ should really be functions of position, since the degree of curve smoothness, and the strength of the observed image gradient, can both be strong functions of location, nevertheless all existing methods considered these parameters to be constant.

Proposed Approach

Proposed approach decomposes the internal and external energy and minimizes individual components which is briefly described by these three iterative steps. Details on each component follow.

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Viterbi Search:

The problem is modeled as a Hidden Markov Model (HMM) and a Viterbi search is used to find the optimal solution by dynamic programming. In the absence of image noise and shape prior, the Viterbi search will identify all of the strongest local boundaries. The trellis for HMM and Viterbi search is shown below.

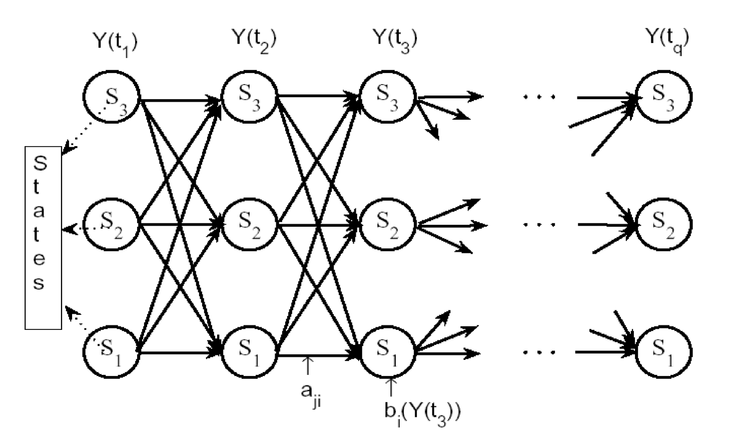

Figure 1: Trellis for hidden Markov model (HMM) and Viterbi search, S_1, S_2 and S_3 are the Hidden states of HMM, $Y(t_1), Y(t_2), \cdots, Y(t_q)$ are the sequence observation of hidden states, $a_j i$ and $b_i(Y(t_3))$ are the state transition probability and emission probability respectively

Viterbi algorithm finds the path that maximizes the joint probability with first order Markov assumption using equation (4) and (5)

$$\delta_t(i) = \Big(\max_j\big(\delta_{t-1}(j)a_{ji}\big)\Big) b_i(Y_t) \tag{4}$$

$$\psi_t(i) = \arg\max\big(\delta_{t-1}(i)a_{ji}\big) \tag{5}$$

Curvature guided importance sampling:

In general, the prior model of objects are defined by constraints on its overall shape. Active contours traditionally consider thin plate and membrane constraints to constitute the prior model of the object which is not an accurate hypothesis for high curvature object boundaries. Therefore, the proposed method generates snake points using importance sampling of the local curvature (K) along the snake which will ensure more samples in high curvature regions using equation (6), (7) and (8)

$$\rho_b \propto \frac{(q_b - q_{b-1})}{(s_b - s_{b-1})} \propto K \tag{6}$$

$$\Delta s_{min} < \Delta s < \Delta s_{max} \tag{7}$$

$$\frac{L}{\Delta s_{\max}} \ll q \ll \frac{L}{\Delta s_{\min}} \tag{8}$$

Statistical estimation:

There are two reasons motivating an estimation step. First, while calculating visual boundary using viterbi, shape priors were not considered, however prior models of shapes play a vital role with high measurement uncertainty. Second, to directly incorporate complicated shape model directly into the Viterbi approach is difficult, even for second order constraint complexity. The new approach computes the object boundary from the observed image ignoring the prior model and then fuses the measured boundary with the prior model statistically using equation (10)

$$M = CZ + W \tag{9}$$

$$\hat{Z} = (C^t R^{-1} C + P^{-1})^{-1} C^t R^{-1} M \tag{10}$$

$$P = \lambda(\pounds\pounds^t)^{-1} \tag{11}$$

$$\pounds = [\alpha(\mathbf{L_x} + \mathbf{L_y}) + \beta(\mathbf{L_{xx}} + \mathbf{L_{yy}})] \tag{12}$$

Where L_x, L_y, L_{xx} and L_{yy} are elastic and membrane constrains respectively The complete algorithm is demonstrated in Figure. 2 and algorithm. 1

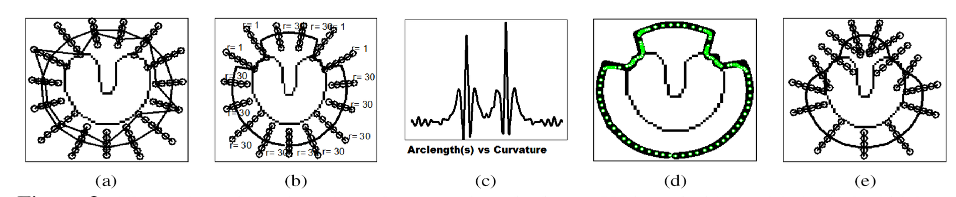

Figure 2: Illustration of one iteration of proposed method on a U-shaped object. (a) A circle (thick line) shows the initial positions of the snake, the jagged line shows a potential snake solution, and the small circles shows the nodes of Viterbi trellis. (b) The thick line shows the optimal snake after a Viterbi search. (c) Curvature of the curve obtained using Viterbi search (X-axis Arc length, Y-axis Curvature(K)). (d) small green circles are the particles generated using importance sampling on curvature of optimal Viterbi snake. (e) The thick line shows the estimated snake as the initial snake to start the next iteration.

Algorithm 1 Pseudo code of proposed method

```
t = 0, e = ∞, ϵ = 1^(−10)
Initialize object boundary manually as v_00(s)
while e ≥ ϵ do
    Generate discrete search space u_ab normal to v_t0(s)
    Find the best curve z_âb using Viterbi search
    Compute curvature of z_âb and perform importance sampling on it to generate more sample near high curvature region. Consider each sample as measurements(M)
    Calculate ẑ(t)
    e = ||ẑ(t) − ẑ(t − 1)||
    v_(t+1)0(s) = ẑ(t)
    t = t + 1
end while
```

Experimental Results and Discussion

To authenticate the capability of the proposed method, testing has been conducted on both original and published images. Two synthetic binary images are tested, one with a V shaped object and the other with a heart shaped object. The four standard images are brain-web, disc on a complex background, starfish, thin u-shape. The labels A, B, C, D, E and F are assigned to V-shape, heart shape, ball in complex background, brain, starfish and thin u-shape images, respectively, as illustrated in Fig. 2. Experiments are performed on 2.61 GHZ, 2GB DD RAM, AMD Athlon 64X2 dual core machine .

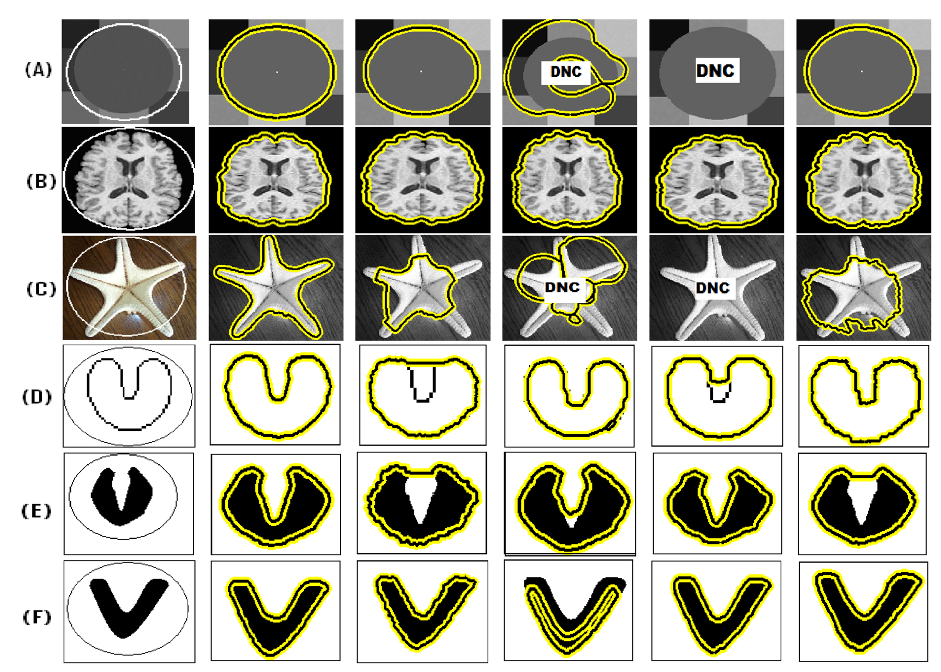

Figure 3: Column 1 the snake generated using the proposed method; column 2 to 5 shows results using four other snake techniques. Yellow lines are final contours. First five rows (A, B, C and D) show images obtained from. Last two rows (E and F) show two synthetically generated images. The proposed method is the only one that can properly identify the object boundary in each case. No other method works for more than four images. Some test had solutions but did not converge to right solutions and some solutions were unbounded (DNC)

Results are shown in Figure. 3 and Table. 1. Figure. 3 illustrates separate images for the initial contour, the solution for the proposed method, and the solution for the four other methods. Table. 1 shows the quantitative values of Mean Square Error (MSE) and Execution Time (ET) of all five methods. In the proposed method, parameters of the discrete snake are guided by curvature and external force. As a result, the proposed method works effectively for all six images without adjusting any parameters. In contrast, the four comparative methods are sensitive to fixed parameters and no other method can effectively identify the necessary boundary for all test images.

Table 1. Comparison table showing Mean Square Error(MSE) and Execution Time (ET) in second of Proposed Method against four other methods for different images. Text in bold letter indicates best performance among their peers for a particular image

	Proposed Snake		Traditional Snake		DTF Snake		BF Snake		GVF Snake	
	MSE	ET	MSE	ET	MSE	ET	MSE	ET	MSE	ET
(A)	**0.89**	**14**	1.4	91	110	42	1.4	93	1.1	112
(B)	**1.1**	**29**	2.9	91	2.6	36	1.2	93	2.5	104
(C)	**1.5**	**4.5**	1.55	36	DNC	38	DNC	70	1.53	52
(D)	**1.8**	**3.5**	1.83	47	2.1	46	1.9	42	1.85	73
(E)	**2.1**	**26**	42	145	DNC	40	DNC	119	351	171
(F)	**2.7**	**31**	12	91	16	48	9.5	94	6	114

Sensitivity to Initial position:

Figure. 4 shows three different initial snake contours and their successful convergence using the proposed method. In contrast, the balloon force (BF) snake requires that the initial snake be placed fully within the solution boundary. Also, the Traditional snake requires an initial snake close to its solution to encourage speed of convergence and accuracy.

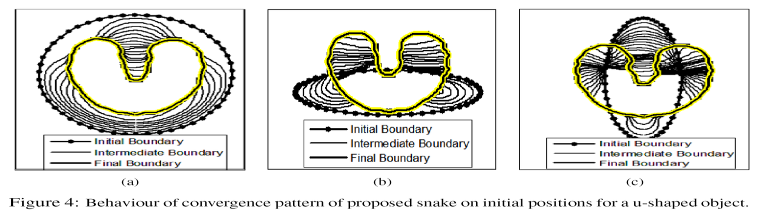

Figure 4: Behaviour of convergence pattern of proposed snake on initial positions for a u-shaped object.

Sensitivity to parameter

Performance of existing contour extraction techniques are dependent upon appropriate values of parameters such as α, β, and γ. However, the proposed method adaptively chooses parameters from curvature and image gradient according to the relation mentioned in equation (13).

$$[\alpha, \beta, \lambda] \propto \rho \propto \frac{1}{r_{bb}} \tag{13}$$

Sensitivity to noise

Figure 5 compares the proposed method to the other four methods for a PSNR of 6 given a noisy image with a diamond shape. Clearly, the proposed method is robust to noise relative to its peers since the proposed method is the only method to successfully identify the diamond.

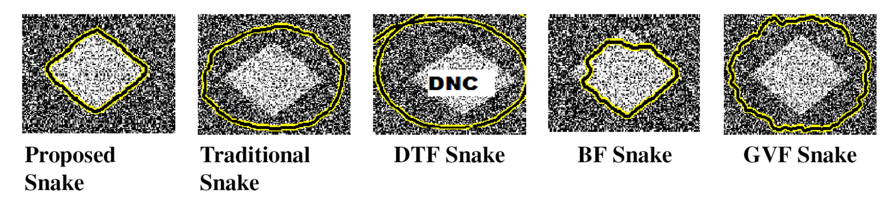

Figure 5: Performance of each method in presence of noise with $\sigma = 0.65$, $\mu = 0.1$, and PSNR = 6. Only the proposed method works effectively in the noisy image.

Conclusion and Future Work

A novel discrete snake for accurate boundary extraction regardless of noise and geometry of the object boundary is designed and implemented. This discrete snake adjusts its parameters iteratively as a function of the current snake solution. The method is demonstrated to be robust to initial parameter setting regardless of the nature of the image. Convergence of proposed method is guaranteed empirically and robustness to noise is shown experimentally. In the future, parametric testing will be conducted to better understand the contribution of each parameter to the final solution.

Acknowledgement

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