Data Fusion of SSM/I Channels using Multiresolution Wavelet Transform

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TABLE I
SIZES OF THE 3-dB ANTENNA FOOTPRINTS AND THE APPROXIMATE SPACING OF THE MEASUREMENTS IN THE TRACK AND SCAN DIRECTIONS OF THE SSM/I CHANNELS (TAKEN FROM [1]).

<table>
<thead>
<tr>
<th>SSM/I Channel</th>
<th>3 dB Footprint (km)</th>
<th>Approximate Spacing (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (GHz)</td>
<td>Pol.</td>
<td>Track</td>
</tr>
<tr>
<td>19.35</td>
<td>V</td>
<td>69</td>
</tr>
<tr>
<td>19.35</td>
<td>H</td>
<td>69</td>
</tr>
<tr>
<td>22.235</td>
<td>V</td>
<td>60</td>
</tr>
<tr>
<td>37.0</td>
<td>V</td>
<td>37</td>
</tr>
<tr>
<td>37.0</td>
<td>H</td>
<td>37</td>
</tr>
<tr>
<td>85.5</td>
<td>V</td>
<td>15</td>
</tr>
<tr>
<td>85.5</td>
<td>H</td>
<td>15</td>
</tr>
</tbody>
</table>

The sampling pattern is such that there is 12.5 km pixel spacing at 85 GHz and 25 km pixel spacing at the lower frequencies. Our algorithm fuses these different frequency data sets in a statistically optimal way, thereby achieving an enhanced spatial resolution while preserving the coverage. We utilize the statistics intrinsic to the field for this purpose. Moreover we then precondition the field to be estimated with the Haar wavelet, which results in computational efficiency and compatibility with FPGA hardware.

II. PROBLEM FORMULATION

The channel measurements are given by:

\[ Y = GX + E, \]  

(1)

where G corresponds to the antenna gain function for all the channels, X is the actual antenna temperature field, and E is noise. Solving the least squares problem in a Bayesian framework and assuming gaussian distributions for X and E we have the estimation of X as:

\[ \hat{X} = (P^+ \cdot G^T R^{-1} G)^{-1} G^T R^{-1} Y, \]  

(2)

where P and R are the \textit{a priori} covariance matrices of X and E respectively, which have to be empirically estimated. By effectively preconditioning the estimation problem using a wavelet transform W we have,

\[ \hat{X} = WX, \]  

(3)

and the corresponding estimation of X is:

\[ \hat{X} = ((WPW^T)^{-1} + (GW^T \cdot R^{-1} GW^T)^{-1}) (GW^T \cdot R^{-1} Y), \]  

(4)

With a suitable choice of a wavelet several simplifications are possible and otherwise one relies on matrix inversions performed off-line (ahead of time). Alternatively a filter may be applied to the estimated field to make it as uncorrelated as
possible so that the prior covariance matrix of the field \( X \) is Identity. This can lead to further simplifications.

III. PRACTICAL CONSIDERATIONS

We consider the actual antenna temperature field \( X \), to live in a grid that is 4 times finer in horizontal and vertical directions than the highest resolution (12km for 85Ghz) of the available measurements. Thus the pixel size for \( X \) is taken to be 3.125x3.125 (km sq).

The gain pattern for each footprint corresponding to each frequency is generally oval in shape and can be approximated by a jointly binomial pattern. For example consider the 85Ghz footprint. It has length of 13 km along scan and 15 km along track. This can be approximated with a 4x6 group of pixels in the finer scale of \( X \). Fig.1(a) shows the approximated 85GHz footprint with total gain normalized to unity. Footprints for other frequencies are similarly obtained. The footprints for different polarizations of the same frequency are assumed to be similar.

The data in both the coarser and finer grids are vectorized into a column vector. The gain matrix \( G \) for a single channel measurement will then transform the finer grid to the coarser grid based on the antenna footprint for that channel. For instance the gain matrix will convert a 4N\times4N field in the finer grid of \( X \) to an \( N\times N \) field in the coarser grid of the 85 GHz channels (for channels other than 85 GHz, the coarser grid will be \( N/2 \times N/2 \)). The overall gain matrix can be generated on this basis.

IV. ESTIMATING THE PRIOR COVARIANCE MATRICES

The empirical statistics of the finest scale are used to obtain a prior covariance model of the field to be estimated (\( X \)). We observe the pixels of 85Ghz in the direction along scan. This direction has minimum overlap between pixels and hence serves as a good approximation to the actual field \( X \). The portions of the 85GHz observed images are generally non-stationary with respect to the mean. Hence mean removal is carried out over windows before constructing the covariance matrices. Fig.2(a) shows the experimentally obtained covariance matrix from the 85GHz, with mean removal done over window sizes of 8x8. The anti-diagonal of the covariance matrix is found to nicely fit an exponential model as shown in Fig.2(b).

With the exponential fit, and assumption of Isotropy, it is then straightforward to construct the covariance matrix \( P \) for the finer field \( X \). This was carried out and is shown in Fig.3 for a field vectorized into a column vector.

With the assumption that all measurement errors are uncorrelated, the corresponding Error covariance Matrix \( R \) is diagonal. Further assuming that for a particular channel all errors have equal variance, we have:

\[
R = \sigma^2 I, \quad (5)
\]

For a particular channel the error variance can be interpreted as a weighting factor for that channel. Hence by keeping higher error variances for the lower resolutions (channels other than 85Ghz) we obtain a method for weighting the various channels. In other words, by assigning lower error variances to the 85GHz channels, we determine our estimated field more from the higher resolution data (that the 85Ghz channels provide) than the lower resolution data.

V. ESTIMATION WITH TWO CHANNELS

A. Direct Method

Consider the 85-H and the 37-H channels. For estimating the finer field based on the data of these two fields, we stack the data one above the other in a column vector and develop the corresponding gain and error covariance matrices for the combined data. We solve the estimation problem for two cases. In the first case we solve for the finer field \( X \) using (2),...
without preconditioning the estimated field by the wavelet transform. The estimated field for this image is obtained as shown in Fig.4.

![Fig.4](image)

Note that for performing these computations the size of the matrices is of the order of NxN, where N is the total number of pixels in the reconstructed field (N is equal to 1600 for the reconstructed field shown in Fig.4).

**B. Estimation using Wavelet Transform:**

By carrying out the cholesky factorization of the prior covariance matrix P of X, and taking the inverse of it, we effectively obtain a filter that makes the field X uncorrelated. That is by defining:

\[ F^{-1} = A, \quad \text{and} \quad AA^T = P, \]  

we have a filter F on X such that:

\[ Y = GF^{-1}X_f + E, \quad \text{where} \quad X_f = FX, \]  

Thus the estimation problem now changes to:

\[ \hat{X}_f = (I + \frac{1}{2} \frac{G_f G_f^T R^{-1} R^{-1} G_f^T}{(G_f^T G_f)^{-1}} G_f^T R^{-1} Y, \]  

and I is the Identity matrix.

Now with the prior covariance matrix of X, being the Identity matrix we precondition X' with the wavelet transform W_l where W_l gives the low frequency portion of the Haar wavelet W. Thus the estimation problem now is:

\[ \hat{X} = (I + \frac{1}{2} \frac{G_l G_l^T R^{-1} R^{-1} G_l^T}{(G_l^T G_l)^{-1}} G_l^T R^{-1} Y, \]  

where \( \hat{X} = W_l X_f \), and \( \bar{G} = G_l W_l^T \),

with the prior covariance matrix on the preconditioned X' still being the Identity matrix but now of size (NxN)/2 where N is the total number of pixels in the patch of field X. Thus we reduce the size of the matrices by a factor of 4 and increase the speed of computations, by removing the high frequency part of the Haar wavelet W. The size of the estimated data \( \hat{X} \) is also (NxN)/2. The actual field that is of size NxN is reconstructed by the following transformation, which can be performed at the ground.

\[ X = F^{-1} W_l^T \hat{X}, \]  

This method when performed for the same block of data yields almost exactly the same results, which means that the high frequency component of the wavelet transform that we removed is not significant.

**VI. CONCLUSION**

This paper demonstrates how data fusion of multiresolution data can be carried out in a statistically optimal way. With experiments carried out on data from SSM/I channels we have shown that the computations can be carried out efficiently by employing the Haar wavelet with results that are similar to those without the preconditioning.

**REFERENCES**


