Deblocking of Block-Transform Compressed Images Using Phase-Adaptive Shifted Thresholding

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Abstract

Many popular image compression schemes are based on block-transform coding, a technique where images are broken into small blocks of pixels prior to transformation and compression. Block-transform coding often introduces blocking artifacts which are particularly prevalent at low bit-rates due to quantization errors. A novel algorithm for deblocking block-transform compressed images is proposed in this paper. This algorithm is based on a phase-adaptive, shifted thresholding technique that estimates the original uncompressed image as the weighted sum of shifted versions of the decompressed image subjected to a threshold. An efficient integer transform is used to construct the shifted versions of the decompressed image. The aggregation weights are obtained adaptively using the local phase moment characteristics of the underlying image content. The proposed algorithm utilizes important human perceptual characteristics to provide effective image deblocking while preserving image detail. Experimental results show that the proposed algorithm is more efficient than comparable methods and yields both subjective results and peak signal-to-noise ratio (PSNR) results comparable to existing methods.

1. Introduction

Image compression has become a very important field of research due to the increasing demand for digital media storage in applications such as remote sensing, digital photography, medical imaging, and entertainment. Image compression is also vital for reducing data transmission requirements of image and video content, which is particularly important given the ever increasing usage of image and video content over the Internet. Thus, considerable effort has gone into the study of image compression algorithms.

Numerous algorithms have been proposed for the purpose of image compression. These can generally be divided into two groups: lossless compression algorithms and lossy compression algorithms. In lossless image compression algorithms, an image is compressed in such a way that all of the original image information is preserved. Typically, such algorithms provide relatively low levels of compression performance. The majority of research effort in recent years has been focused on lossy image compression algorithms, where image information is lost during the image compression process. Lossy image compression algorithms can be used to achieve very high levels of image compression performance at the expense of image quality. The most widely used lossy image compression algorithms are those based on block-transform coding (BTC), where images are divided into small blocks of pixels prior to transformation and compression. BTC compression algorithms have low computation and storage requirements due to the processing of image content in small (independent) blocks. Widely-used BTC compression algorithms include JPEG [1], MPEG [2, 3], and H.264 [4].

A major problem encountered when using BTC compression algorithms is the introduction of blocking artifacts at block boundaries. This is due to the fact that the blocks of the image are processed independently of each other. As such, quantization errors that occur during the lossy compression process become very noticeable at the block boundaries, thus reducing the perceived image quality of the compressed image. Blocking artifacts are most often observed at low bit-rates where quantization errors become visually prominent. Therefore, a method to reduce such blocking artifacts is desired.

A large number of algorithms have been proposed for reducing blocking artifacts in images compressed using BTC compression algorithms. These deblocking algorithms can be generally organized as follows:

1. Projections onto convex sets (POCS) methods [5, 6]

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2. Spatial block boundary filtering methods [7, 9]
3. Wavelet filtering methods [10]
4. Statistical modeling methods [11]
5. Constrained optimization methods [12], and
6. Shifted transform methods [13, 15, 14, 16]

Of the aforementioned algorithms, those based on shifted transforms have been shown to be highly effective at reducing blocking artifacts. Originally introduced by Nosratinia for JPEG images [13], the concept of shifted transforms has been subsequently extended for wavelet compression [15] as well as for reducing ringing artifacts [14]. However, the computational effort typically associated with shifted transform methods has prevented their widespread use in many applications. Recently, a computationally efficient variation of a shifted transform deblocking algorithm was introduced by Wong et al. that was shown to require significantly less computational effort while providing comparable visual quality improvements to that proposed by Nosratinia [16]. A hardware implementation of this algorithm has subsequently been introduced to provide real-time video deblocking in an embedded system. [17].

A drawback of deblocking algorithms based on shifted transforms is that they do not take advantage of the perceptual characteristics of the underlying image content in an adaptive manner. Since all image content is treated in a fixed manner, perceptually important characteristics of the image may be degraded or lost during the deblocking process. The goal of this paper is to address this issue by adapting the deblocking process using the perceptual characteristics of the underlying image content such that perceptually important detail is preserved.

The main contribution of this paper is an algorithm for deblocking block-transform compressed images based on phase-adaptive shifted thresholding. The proposed algorithm is efficient and utilizes the phase moment characteristics of the underlying image content to adapt the image deblocking process to better preserve image detail. The proposed method is described in Section 2. Experimental results are presented and discussed in Section 3. Finally, conclusions are drawn in Section 4.

2. Proposed Deblocking Algorithm

The proposed deblocking algorithm utilizes the concept of local phase characteristics and shifted transforms in an attempt to better preserve perceptually important detail in the underlying image content. The proposed method can be briefly described as follows. First, shifted versions of the decompressed image are constructed using an efficient integer transform and are subjected to a threshold. Second, the local phase moment characteristics are obtained from the set of images. Finally, the deblocked image is estimated as the weighted sum of the set of images using a phase-adaptive aggregation scheme. An overview of the proposed algorithm is shown in Figure 1.

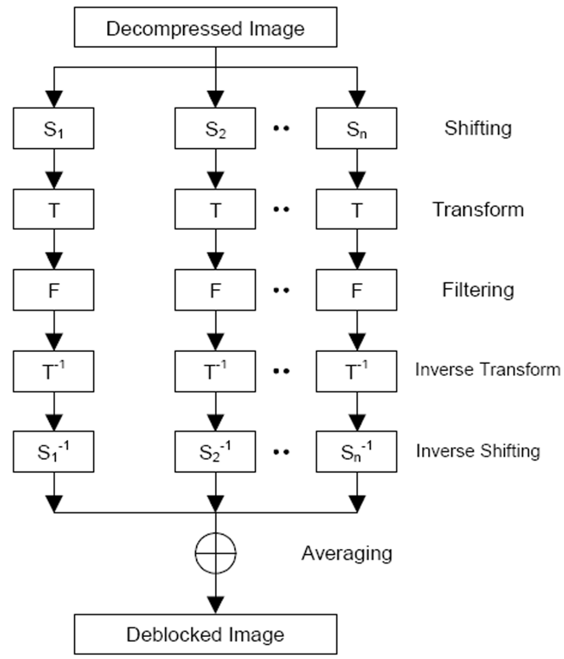

Figure 1. Overview of the proposed algorithm

2.1. Shifted Thresholding

The first step in the proposed algorithm is to construct shifted versions of the decompressed image. Given a decompressed image I_0, a set of four shifted images $\{I_1, I_2, I_3, I_4\}$ is constructed from I_0 based on the following shifts:

$$(S_x, S_y) = \{(-3,-3), (-1,-1), (1,1), (3,3)\} \quad (1)$$

where S_x and S_y represent shifts in the x and y directions, respectively. This set of shift patterns has been shown to provide good deblocking performance while requiring significantly fewer computations than methods such as those proposed by Nosratinia [16]. The shifting process is shown in Figure 2.

Once the set of shifted images $\{I_1, I_2, I_3, I_4\}$ have been constructed, they are transformed into the spatial frequency domain using the efficient integer transform proposed in [16]. This integer transform T is a scaled integer

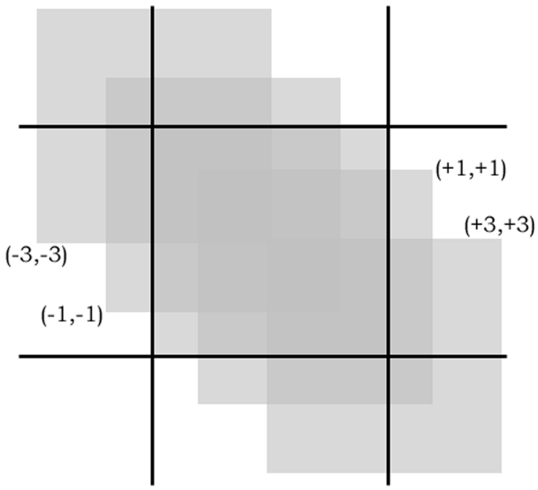

Figure 2. The shifting process. Four different shifted images are constructed based on four different shifting patterns.

approximation of the Discrete Cosine Transform (DCT) and can be expressed as follows:

$$\mathrm{T} = \mathrm{round}(\alpha \mathrm{F}) \tag{2}$$

where F is the DCT transform matrix and α is the scaling factor. For the proposed algorithm, $\alpha = 512$ and so T for the case of 8×8 blocks is computed as:

$$\mathrm{T} = \begin{bmatrix} 181 & 181 & 181 & 181 & 181 & 181 & 181 & 181 \\ 256 & 206 & 128 & 64 & -64 & -128 & -206 & -256 \\ 256 & 64 & -64 & -256 & -256 & -64 & 64 & 256 \\ 206 & -64 & -256 & -128 & 128 & 256 & 64 & -206 \\ 181 & -181 & -181 & 181 & 181 & -181 & -181 & 181 \\ 128 & -256 & 64 & 206 & -206 & -64 & 256 & -128 \\ 64 & -256 & 256 & -64 & -64 & 256 & -256 & 64 \\ 64 & -128 & 206 & -256 & 256 & -206 & 128 & -64 \end{bmatrix} \tag{3}$$

The transformation of the i^{th} shifted image I_i into the spatial frequency domain using the integer transform T can be expressed as follows:

$$\mathrm{J_i} = \left(\mathrm{T I_i T^T}\right) >> \left(\log_2\left(\alpha^2\right)\right) \tag{4}$$

where T and $\mathrm{T^T}$ is the integer transform and its transpose respectively and $>>$ represents the bit shift operation. The number of multiplications required to perform the integer transform can be further reduced by decomposing T into a scalar product of two simpler matrices as follows:

$$\mathrm{T} = \mathrm{U} \otimes \mathrm{V} \tag{5}$$

Based on this decomposition, the integer transform can be written as follows:

$$\begin{aligned} \mathrm{J_i} &= (\mathrm{U} \otimes \mathrm{V})\, \mathrm{I_i}\, (\mathrm{U} \otimes \mathrm{V})^{\mathrm{T}} >> \left(\log_2\left(\alpha^2\right)\right) \\ &= \left(\mathrm{U I_i U^T}\right) \otimes \left(\mathrm{V} \otimes \mathrm{V^T}\right) >> \left(\log_2\left(\alpha^2\right)\right) \end{aligned} \tag{6}$$

where U and $\mathrm{V} \otimes \mathrm{V^T}$ are defined as follows for the case of 8×8 blocks and $\alpha = 512$:

$$\mathrm{U} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 256 & 206 & 128 & 64 & -64 & -128 & -206 & -256 \\ 256 & 64 & -64 & -256 & -256 & -64 & 64 & 256 \\ 206 & -64 & -256 & -128 & 128 & 256 & 64 & -206 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 128 & -256 & 64 & 206 & -206 & -64 & 256 & -128 \\ 64 & -256 & 256 & -64 & -64 & 256 & -256 & 64 \\ 64 & -128 & 206 & -256 & 256 & -206 & 128 & -64 \end{bmatrix} \tag{7}$$

$$\mathrm{V} \otimes \mathrm{V^T} = \begin{bmatrix} 32761 & 181 & 181 & 181 & 32761 & 181 & 181 & 181 \\ 181 & 1 & 1 & 1 & 181 & 1 & 1 & 1 \\ 181 & 1 & 1 & 1 & 181 & 1 & 1 & 1 \\ 181 & 1 & 1 & 1 & 181 & 1 & 1 & 1 \\ 32761 & 181 & 181 & 181 & 32761 & 181 & 181 & 181 \\ 181 & 1 & 1 & 1 & 181 & 1 & 1 & 1 \\ 181 & 1 & 1 & 1 & 181 & 1 & 1 & 1 \\ 181 & 1 & 1 & 1 & 181 & 1 & 1 & 1 \end{bmatrix} \tag{8}$$

The matrices U, $\mathrm{U^T}$, and $\mathrm{V} \otimes \mathrm{V^T}$ are pre-computed for efficiency and stored since they are used for all block transforms. Another key benefit to the above formulation is that a majority of the multiplications can be performed using bit-shift operations that are significantly faster than normal multiplication operations.

Once the set of transformed images $\{F_1, F_2, F_3, F_4\}$ have been constructed, the images are subjected to a threshold in the spatial frequency domain to produce $\{F'_1, F'_2, F'_3, F'_4\}$ based on a set of frequency thresholds P as defined by:

$$\mathrm{P} = \mathrm{Q} >> 1 \tag{9}$$

where Q is the set of quantization parameters pertaining to the decompressed image. The set of quantization parameters are obtained as the scaled integer approximation of the quantization parameters stored with the image that are used to decompress the image. This thresholding process is designed to eliminate the high frequency characteristics associated with blocking artifacts along the block boundaries for each of the shifted images. By using the quantization parameters pertaining to the decompressed image as the basis for the frequency thresholds, the thresholding process is adaptively adjusted based on prior information about the compression process to achieve better deblocking performance.

After the thresholding process, an inverse integer transform is performed on the set of images $\{F'_1, F'_2, F'_3, F'_4\}$ as follows:

$$\mathrm{I'_i} = \left(\mathrm{U^T}\left(\mathrm{J'_i} \otimes \left(\mathrm{V} \otimes \mathrm{V^T}\right)\right) \mathrm{U}\right) >> \left(\log_2\left(\alpha^2\right)\right) \tag{10}$$

Finally, the shifted images $\{I'_1, I'_2, I'_3, I'_4\}$ are inversely shifted to obtain $\{I''_1, I''_2, I''_3, I''_4\}$ based on the following shifts:

$$(S^{-1}{}_x, S^{-1}{}_y) = \{(3,3), (1,1), (-1,-1), (-3,-3)\} \tag{11}$$

2.2. Local Phase Moment Extraction

One of the main objectives of the proposed deblocking algorithm is to improve the perceptual quality of the deblocked image adaptively based on the perceptual characteristics of the underlying image content. Therefore, a measure for human perceptual significance pertaining to visual information is required. A particularly effective approach for measuring and modeling perceptual characteristics is that based on local phase characteristics [18, 19, 20, 21, 22, 23]. Of particular importance with respect to measuring perceptual significance is local phase coherence, which has been postulated to be maximal at locations pertaining to perceptually significant characteristics in an image [18]. The correspondence of local phase coherence to perceptual significance has been further reinforced by psychophysical evidence [19]. Furthermore, this measure of perceptual significance is invariant to global and local contrast conditions. Given all these benefits, the proposed deblocking algorithm utilizes local phase coherence information to adapt the deblocking process.

To derive the local phase coherence information from the underlying image content, the local amplitude and phase information is extracted at different scales and orientations from I''_1 using complex wavelets (i.e., Gabor wavelets, Dual-tree wavelets, etc.) as follows:

$$A_n(\underline{x}) = \sqrt{(I''_1(\underline{x}) * F^e_n)^2 + (I''_1(\underline{x}) * F^o_n)^2} \tag{12}$$

$$\phi_n(\underline{x}) = \tan^{-1}\left(\frac{(I''_1(\underline{x}) * F^e_n)}{(I''_1(\underline{x}) * F^o_n)}\right) \tag{13}$$

where F^e_n and F^o_n are the even-symmetric and odd-symmetric wavelets at scale n and $\underline{x} = (x, y)$. Based on the extracted local amplitude and phase information, the local phase coherence κ is computed as follows [22]:

$$\kappa(\underline{x}, \theta) = \frac{\sum\limits_n W(\underline{x}, \theta) \left\lfloor A_n(\underline{x}, \theta) \Delta\Phi(\underline{x}, \theta) - T \right\rfloor}{\sum\limits_n A_n(\underline{x}, \theta) + \varepsilon} \tag{14}$$

$$\begin{aligned}\Delta\Phi(\underline{x}, \theta) = \quad & \cos\left(\phi_n(\underline{x}, \theta) - \bar{\phi}(\underline{x}, \theta)\right) \\ & - \left|\sin\left(\phi_n(\underline{x}, \theta) - \bar{\phi}(\underline{x}, \theta)\right)\right|\end{aligned} \tag{15}$$

where W represents the frequency spread weighting factor, A_n and ϕ_n are the local amplitude and phase at wavelet scale n respectively, $\bar{\phi}$ is the weighted mean phase, T is the noise threshold and ε is a small constant to prevent division by zero.

The measure for perceptual significance of the underlying image content can then be determined as the maximum moments of phase coherence ρ across all orientations:

$$\rho(\underline{x}) = \frac{1}{2}\left(\begin{array}{l}\sum\limits_\theta \left[\left(\kappa(\underline{x}, \theta)\right)^2\right] + \\ \sqrt{\begin{array}{l} 4\left(\sum\limits_\theta \left(\kappa(\underline{x}, \theta)\sin(\theta)\right)\left(\kappa(\underline{x}, \theta)\cos(\theta)\right)\right)^2 + \\ \left(\sum\limits_\theta \left[\left(\kappa(\underline{x}, \theta)\cos(\theta)\right)^2 - \left(\kappa(\underline{x}, \theta)\sin(\theta)\right)^2\right]\right)^2 \end{array}}\end{array}\right), \tag{16}$$

where $\kappa(\underline{x}, \theta)$ is the local phase coherence at orientation θ. High values of ρ indicate high perceptual significance in the underlying image content.

2.3. Phase-Adaptive Weighted Aggregation of Reconstructed Images

With the set of images $\{I''_1, I''_2, I''_3, I''_4\}$ and perceptual significance measure ρ in place, it is necessary to estimate the original image by aggregating the set of images in such a way that perceptual detail is well preserved. In the proposed deblocking algorithm, the original image $\hat{I}_d(\underline{x})$ is estimated as the weighted sum of decompressed image I_0 and the corresponding set of images $\{I''_1, I''_2, I''_3, I''_4\}$ as follows:

$$\hat{I}_d(\underline{x}) = \frac{\sum\limits_i w_i(\underline{x})\, I_i(\underline{x})}{\sum\limits_i w_i(\underline{x})} \tag{17}$$

where w_i represents the weight associated with the i_{th} image. It can be seen that the selection of aggregation weights has a significant impact on the quality of the resulting image estimate. In the proposed method, the aggregation weights are adaptively adjusted based on the local phase moment characteristics of the underlying image content. The scaling of aggregation weights can be adjusted to either preserve image characteristics by using large weights, or provide a smoother estimate of image characteristics by using smaller weights. For the purpose of enhancing perceptual quality, the weights of the decompressed image I_0 are increased to avoid over-smoothing at pixels with high perceptual significance (as indicated by high values of ρ). At the same time, it is important to suppress blocking artifacts in areas with low perceptual significance as they are noticeable in such areas. Therefore, increasing the weights of the set of images $\{I''_1, I''_2, I''_3, I''_4\}$ in such areas provides improved blocking artifact reduction. Based on the aforementioned motivations, the aggregation weights are adaptively computed using the following expression:

$$w_i(\underline{x}) = \begin{cases} \alpha + (\beta - \alpha)\,\rho(\underline{x}), & i = 0 \\ (1 - \alpha + (\beta - \alpha)\,\rho(\underline{x}))/4, & i > 0 \end{cases} \tag{18}$$

where α and β are the minimum and maximum weight constraints for I_0. During testing, it was found that good deblocking performance and detail preservation can be obtained using $\alpha = 0.1$ and $\beta = 0.9$. The main advantage of the proposed adaptive weighted aggregation scheme is that it allows for the system to find a balance between blocking artifact reduction and perceptual detail preservation in an adaptive and fine-grained manner based on the perceptual characteristics of the underlying image content.

3. Experimental Results

To investigate the effectiveness of the proposed algorithm, a comparative analysis was performed between the proposed algorithm, the method proposed by Nosratinia [13, 15] and the method proposed by Wong et al. [16]. The deblocking algorithms were tested using the 5 different test stock images shown in Figure 3.

Figure 3. Test set of 5 images

To evaluate the performance of the proposed method in a quantitative manner, the peak-signal-to-noise ratio (PSNR) was computed for the test images for the tested algorithms at a fixed compression rate using JPEG, which remains the most widely used still image format, despite the introduction of newer formats. The most widely used measure for image quality assessment, PSNR provides a general picture of how well information from the original image is preserved in the compressed image.

A summary of the PSNR between the original image and the images produced using the tested deblocking algorithms is presented in Table 1. It can be observed that images produced using the proposed method shows noticeable PSNR gains over previous methods for all test images. These results demonstrate the effectiveness of the proposed algorithm in preserving information from the original image.

Some of the test images deblocked using the tested algorithms are shown in Fig. 4 and Fig. 5. It can be observed that the quality of the images produced by the proposed algorithm are noticeably better than the other methods. The edges in the images produced by the proposed method are noticeably sharper and fine details are better preserved in all regions compared to the other methods. Furthermore, the images exhibit significantly fewer blocking artifacts than the Wong method while comparable to the Nosratinia method. This demonstrates the effectiveness of the proposed method in providing effective deblocking performance while preserving perceptual detail.

4. Conclusions

A novel algorithm for deblocking block-transform compressed images based on phase-adaptive shifted thresholding is proposed in this paper. The proposed algorithm estimates the original uncompressed image as the weighted sum of shifted versions of the decompressed image constructed using an efficient integer transform. The aggregation weights are adaptively obtained based on the local phase moment characteristics of the underlying image content. The proposed algorithm is highly efficient and utilizes important human perceptual characteristics to provide effective image deblocking while preserving the perception of image detail. Experimental results show that the proposed algorithm produces good subjective results and PSNR results that are comparable to existing methods.

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Table 1. PSNR for Test Images

Image	PSNR of decompressed image (dB)	PSNR Gain (dB) Nosratinia Method [13, 15]	Wong Method [16]	Proposed Method
LENA	31.13 dB	+1.06	+0.90	+1.39
ELAINE	30.48 dB	+0.69	+0.63	+1.02
MANDRILL	24.06 dB	+0.32	+0.23	+0.35
PEPPERS	31.16 dB	+0.93	+0.83	+1.26
BOAT	28.85 dB	+0.76	+0.62	+1.10

Figure 4. Part of JPEG-compressed "Lena" image (from left to right): a) decompressed image, b) deblocked using the Nosratinia method [13, 15], c) deblocked using the Wong method [16], and d) deblocked using the proposed method.

Figure 5. Part of JPEG-compressed "Elaine" image (from left to right): a) decompressed image, b) deblocked using the Nosratinia method [13, 15], c) deblocked using the Wong method [16], and d) deblocked using the proposed method.

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