QMCTLS: Quasi Monte Carlo Texture Likelihood Sampling for Despeckling of Complex Polarimetric SAR Images

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***Abstract*—Despeckling of complex polarimetric SAR images is more difficult than denoising of general images due to the low signal-to-noise ratio and the complex signals. A novel stochastic polarimetric SAR despeckling technique based on quasi Monte Carlo sampling (QMCS) and region-based probabilistic similarity likelihood has been developed. The despeckling of complex polarimetric SAR images is formulated as a Bayesian least squares optimization problem, where the posterior distribution is estimated by QMCS in a nonparametric manner. The QMCS approach allows the incorporation of the statistical description of local texture pattern similarity. Experiments on two benchmark quad-pol SAR images demonstrate that the proposed QMCTLS filter outperforms referenced methods in terms of both noise removal and detail preservation.**

I. Introduction

Polarimetric SAR imagery is a very important source of information for a wide range of remote sensing applications, including terrain classification, target detection, and sea-ice analysis. However, polarimetric SAR imagery is contaminated by speckle noise, which greatly hampers visual interpretation and computer-aided data analysis. While signal averaging is a common tool used to deal with speckle noise in non-complex SAR magnitude data, the scattering matrix for complex polarimetric SAR data needs to be converted to a covariance or coherency matrix before performing averaging [1]. The simplest method is the boxcar method that performs the filtering in a square window. Lee et al. [2] used a local statistics filter which is applied in edge-aligned nonsquare windows. Vasile et al. [3] proposed an intensity-driven adaptive-neighborhood (IDAN) filter which adopts an adaptive neighborhood formed by a region growing technique. Such local methods are not easy to achieve an optimal balance between noise removal and detail preservation.

In contrast to these local methods, the non-local means (NLM) approaches perform despeckling based on observations that are beyond the traditional local window. In NLM approaches, observations that are similar to the referenced observation in a nonlocal searching area are used for performing weighted averaging. Chen et al. [4] proposed a despeckling method called “Pretest” which uses a likelihood-ratio test statistic based on the complex Wishart distribution to measure the similarity between patches. Liu and Zhong [5] proposed a filter based on discriminative similarity measure under a maximum a posteriori (MAP) framework. Torres et al. [6] proposed a new polarimetric filter based on stochastic distances, hypothesis test and nonlocal means techniques.

In this letter, complex polarimetric SAR despeckling is formulated as a Bayesian least squares optimization problem, where the posterior distribution is estimated non-parametrically by quasi Monte Carlo sampling (QMCS). Previously Monte Carlo sampling approaches have been applied for denoising of natural images [7] and despeckling of single-channel SAR images [8]. However, both approaches are not suitable for the despeckling of complex polarimetric SAR images, considering the particularities of polarimetric SAR data. This letter therefore aims to design a new statistical approach based on QMCS for complex polarimetric SAR image despeckling. Compared with the traditional Monte Carlo sampling, QMCS has faster convergence rate, and thus requires fewer samples to achieve an accurate estimation of the posterior distribution. Moreover, a region-based acceptance likelihood based on the complex Wishart distribution is used to aid the QMCS process. Compared with the pixel-based likelihood, the region-based likelihood is capable of capturing the local textual patterns, and is therefore more robust to speckle noise. After the posterior distribution is obtained, the noise-free image value is estimated as the discrete conditional mean.

II. Methodology

A. Complex Polarimetric SAR Noise Model

The degradation model of multi-look polarimetric SAR images can be formulated as [9]:

$$\mathbf{Z} = \mathbf{C} + \mathbf{N_m} + \mathbf{N_a} \tag{1}$$

where $\mathbf{Z}$ is the observed covariance matrix, $\mathbf{C}$ is the true covariance matrix, $\mathbf{N_m}$ represents a multiplicative speckle noise component, and $\mathbf{N_a}$ represents an additive speckle noise component.

$\mathbf{Z}$ is usually modeled as a complex Wishart distribution [1]:

$$p_Z^{(n)}(\mathbf{Z}) = \frac{n^{qn}\,|\mathbf{Z}|^{n-q}\,exp(-nTr(\mathbf{C}^{-1}\mathbf{Z}))}{\pi^{\frac{1}{2}q(q-1)}\,|\mathbf{C}|^{n}\prod_{i=1}^{q}\Gamma(n-i+1)} \tag{2}$$

where n is the number of looks, q is the number of elements in the covariance matrix, which is equal to 3 under the

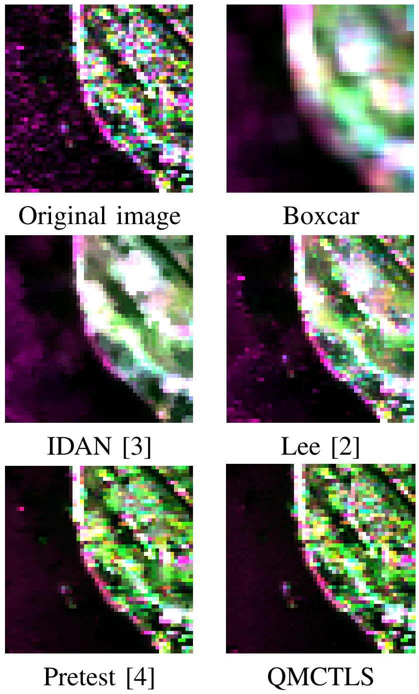

Fig. 1. Zoomed-in of the San Francisco image, containing both sea and urban areas. The proposed QMCTLS filter achieves a smoother result within the homogeneous sea area, and preserves details in the urban area

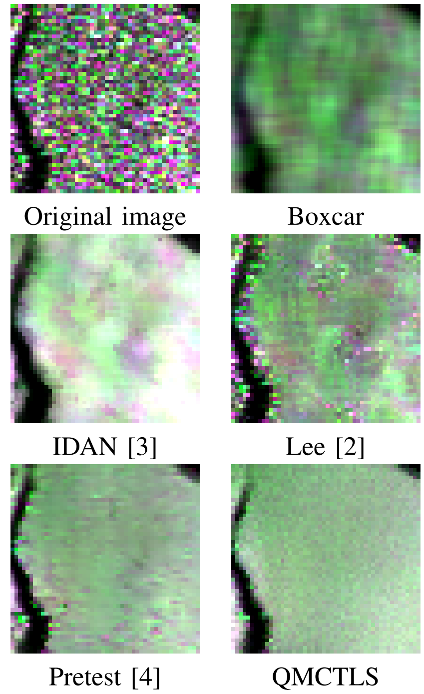

Fig. 2. Zoomed-in of the Ottawa image at rural area. The proposed QMCTLS filter not only preserves river boundary very well, but also achieves smoother results at homogeneous area

reciprocity assumption, $Tr(\cdot)$ is the trace of the matrix, $|\cdot|$ is the determinant function, and $\Gamma(\cdot)$ is the Gamma function.

B. Problem Formulation

Complex polarimetric SAR image despeckling is essentially an inverse problem, where the estimation of noise-free value $\mathbf{C}$ from the observed noisy-value $\mathbf{Z}$ can be formulated as a Bayesian least squares optimization problem [10]:

$$\begin{aligned} \hat{\mathbf{C}} &= \underset{\mathbf{C}}{\operatorname{argmin}} \left\{ E((\mathbf{C}-\hat{\mathbf{C}})^2 \mid \mathbf{Z}) \right\} \\ &= \underset{\mathbf{C}}{\operatorname{argmin}} \left\{ \int (\mathbf{C}-\hat{\mathbf{C}})^2 p(\mathbf{C} \mid \mathbf{Z}) d\mathbf{C} \right\} \end{aligned} \tag{3}$$

In the above equation, $p(\mathbf{C} \mid \mathbf{Z})$ represents the posterior distribution of $\mathbf{C}$ given $\mathbf{Z}$. To estimate $\hat{\mathbf{C}}$, the derivative is taken and set to zero:

$$\frac{\partial}{\partial \hat{\mathbf{C}}} \int (\mathbf{C}-\hat{\mathbf{C}})^2 p(\mathbf{C} \mid \mathbf{Z}) d\mathbf{C} = \int 2(\mathbf{C}-\hat{\mathbf{C}}) p(\mathbf{C} \mid \mathbf{Z}) d\mathbf{C} = 0 \tag{4}$$

$$\int \mathbf{C} p(\mathbf{C} \mid \mathbf{Z}) d\mathbf{C} = \int \hat{\mathbf{C}} p(\mathbf{C} \mid \mathbf{Z}) d\mathbf{C} \tag{5}$$

The right-hand side of (5) can be simplified as:

$$\int \hat{\mathbf{C}} p(\mathbf{C} \mid \mathbf{Z}) d\mathbf{C} = \hat{\mathbf{C}} \int p(\mathbf{C} \mid \mathbf{Z}) d\mathbf{C} = \hat{\mathbf{C}} \tag{6}$$

Therefore, the goal is to estimate the conditional mean:

$$\hat{\mathbf{C}} = \int \mathbf{C} p(\mathbf{C} \mid \mathbf{Z}) d\mathbf{C} = E(\mathbf{C} \mid \mathbf{Z}) \tag{7}$$

However, estimating the conditional mean directly is difficult because the posterior distribution $p(\mathbf{C} \mid \mathbf{Z})$ is unknown. In Section II-D, $p(\mathbf{C} \mid \mathbf{Z})$ is estimated via stochastic QMCS, based on the texture similarity likelihood in Section II-C. The weighted histogram, as the estimate of $p(\mathbf{C} \mid \mathbf{Z})$, will afterwards be used to calculate $\hat{\mathbf{C}}$, as the discrete conditional mean.

C. Region-based Texture Similarity Likelihood

The QMCS approach requires a probabilistic measure of the similarity between two patches. A probabilistic similarity likelihood can be derived based on the log-likelihood-ratio test statistic for equality of two observed complex Wishart matrices $\mathbf{Z_0}$ and $\mathbf{Z_k}$ with equal number of looks in the image [11]:

$$lnQ = n(6ln2 + ln\,|\mathbf{Z_0}| + ln\,|\mathbf{Z_k}| - 2ln\,|\mathbf{Z_0} + \mathbf{Z_k}|) \tag{8}$$

Based on this statistic, the probabilistic similarity likelihood $\alpha(\mathbf{Z_k} \mid \mathbf{Z_0})$ between the two matrices can be expressed as [11]:

$$\begin{aligned} &\alpha(\mathbf{Z_k} \mid \mathbf{Z_0}) = P\left\{-2\rho lnQ \geq z\right\} \simeq 1 - \\ &\omega_2 P\left\{\chi^2(q^2+4) \leq z\right\} - (1-\omega_2) P\left\{\chi^2(q^2) \leq z\right\} \end{aligned} \tag{9}$$

where $\rho = 1 - 17/(12n)$, $\omega_2 = 423/(24n-34)^2$, χ^2 is the chi-square distribution, and z is the observed Q. The term $P\left\{-2\rho lnQ \geq z\right\}$ in (9) represents the probability of finding a bigger value than $-2\rho lnQ$. It measures the probability of similarity between $\mathbf{Z_0}$ and $\mathbf{Z_k}$, and returns a value between 0 and 1. When $\mathbf{Z_k} = \mathbf{Z_0}$, it reaches the maximum.

Therefore, based on the complex Wishart distribution, (9) provides a statistical similarity measure between two pixel-observations in polarimetric SAR images. However, a pixel-based probabilistic measure is incapable of accounting for the local texture patterns, and thus, is very sensitive to speckle noise. Accordingly, in order to utilize the textural information for improving despeckling performance, in this section, a region-based similarity likelihood is introduced to replace the one in (9).

While a region can be assumed an irregular shape, here it is defined as a square window. We assume that the elements in the local region are statistically independent. Accordingly, the texture similarity likelihood of a new region R_k relative to the referenced region R_0 can be formulated as:

$$\alpha(R_k \mid R_0) = \left\{ \prod_j \{P(-2\rho ln Q_j \geq z)\} \right\}^{1/\beta} \tag{10}$$

where β is a scaling parameter ranging from 0 to $+\infty$, and j is used to iterate over all pixels in regions.

D. Posterior Estimation Using Quasi Monte Carlo Sampling

A QMCS strategy is used to estimate the posterior probability $p(\mathbf{C}|\mathbf{Z})$ in a non-parametric manner, which is more flexible than parametric estimation approaches. Moreover, compared to the standard Monte Carlo method that uses a pseudo-random sequence, QMCS uses a low-discrepancy sequence which has a faster rate of convergence [12]. The region-based textural likelihood described in Section II-C is used here to help the sampling process.

In the proposed QMCTLS filter, QMCS is first used to randomly select some pixels around pixel $\mathbf{Z_0}$. Then, an acceptance probability of a sampled pixel $\mathbf{Z_k}$, given the referenced pixel $\mathbf{Z_0}$, is used to decide whether to accept $\mathbf{Z_k}$ or to reject it. The acceptance probability is realized by probabilistic similarity likelihood measures, introduced in Section II-C. Instead of using $\alpha(\mathbf{Z_k} \mid \mathbf{Z_0})$, we use $\alpha(R_k \mid R_0)$ as the acceptance probability of $\mathbf{Z_k}$, in order to account for the local texture patterns. Square regions centered at all the sampled pixels, which we call region-based samples, requires to be obtained to calculate the acceptance probability.

Then, $\alpha(R_k \mid R_0)$ is compared with a random number between $(0, 1)$ drawn from uniform distribution. If $\alpha(R_k \mid R_0)$ is greater than the random number, the center pixel in the sampled region will then be accepted into a sample set Ω. Otherwise, the sample is rejected. After sample pixels are selected into Ω, the posterior distribution is computed in a nonparametric manner via a weighted histogram.

Given samples in Ω, the importance-weighted Monte Carlo posterior estimation can be calculated according to the following weighted histogram [7]:

$$\hat{p}(\mathbf{C} \mid \mathbf{Z_0}) = \frac{\sum_{k\in\Omega} \alpha(R_k \mid R_0)\delta(\mathbf{C} - \mathbf{Z_k})}{N} \tag{11}$$

where $\mathbf{Z_k}$ is the center pixel in region R_k, $\alpha(R_k \mid R_0)$ represents the acceptance probability of $\mathbf{Z_k}$ in Ω, $\delta(\cdot)$ is the Dirac delta function, and N is a normalization term such that $\sum_{\mathbf{C}} \hat{p}(\mathbf{C} \mid \mathbf{Z_0}) = 1$.

Based on $\hat{p}(\mathbf{C} \mid \mathbf{Z_0})$, we can estimate the noise-free image value as the discrete conditional mean, according to (7).

E. Summary of the QMCTLS filter

The proposed QMCTLS filter is processed as the following steps:

1) For each pixel in the image, draw M region-based samples from a search space around the referenced pixel $\mathbf{Z_0}$ using QMCS.

2) For each sample, calculate the probabilistic similarity measure of each pixel pair $R_k(j)$ and $R_0(j)$ at location j of the region, using (8) and (9).

3) Calculate the acceptance probability $\alpha(R_k \mid R_0)$ based on all the pixel pairs using (10).

4) Generate a value u randomly from a uniform distribution $U(0, 1)$. The sample k is accepted into the sample set Ω, if $u \leq \alpha(R_k \mid R_0)$, or otherwise discarded.

5) After processing the M samples by repeating (2)-(4), the accepted samples in Ω are used to estimate the posterior distribution $\hat{p}(\mathbf{C} \mid \mathbf{Z_0})$ in (11);

6) Compute the noise-free estimation $\hat{\mathbf{C_0}}$ of the reference pixel $\mathbf{Z_0}$ as the discrete conditional mean, according to (7).

7) Repeat (1)-(6) for all the pixels in the image.

III. Experiments and Discussion

Two benchmark airborne quad-polarimetric SAR images for testing in comparison with several other popular methods. A 300×300 subimage from the four-look San Francisco image, acquired by AIRSAR in L band with 10×10 m spatial resolution, and the 340×220 ten-look Ottawa image, acquired by a C-band sensor on a Convair 580 platform with 64 cm $\times 10$ m resolution, are used for testing. Methods for comparison include three classical filters, i.e., the Boxcar filter, IDAN filter [3] and refined Lee filter [2], and the state-of-the-art Pretest filter [4]. The window size of IDAN filter is set to be 50×50 pixels, and the size of the refined Lee filter filter is set to be 7×7 pixels. The Pretest filter is set to have a search area of 15×15 and patch size of 3×3. All parameter settings follow the authors' suggestions in the referenced papers.

The QMCTLS filter was implemented in Microsoft Visual C++ 2010. The search area is set to be 21×21, with approximately 50% of the pixels in the search area sampled via QMCS, which is a tradeoff between computation cost and filtering performance. The region is defined as a 5×5 square window. When the window size increases, the acceptance probability is more reliable by considering more pixels, but in the meanwhile the chance of finding similar samples decreases, and the computation cost will increase as well. The scaling parameter β in (10) is set to be the patch size, i.e., 25. The experiments are conducted on an Intel Core i7-4770@ 3.4 GHz. It takes 308 and 265 seconds to filter the San Francisco and Ottawa image respectively.

Fig. 3 and Fig. 4 show the despeckling results in Pauli RGB. They indicate that classical methods, i.e., Boxcar, IDAN and the refined Lee filter, remove speckle noise in stationary areas, but tend to erase details in heterogenous areas. The reason that these classical methods fail to achieve a good balance between noise removal and detail preservation is probably because they rely on local heterogeneity measures, which could not reflect true texture patterns. The Pretest method preserves more details in the urban areas by using non-local information. QMCTLS is comparable to the Pretest method based on visual evaluation. Fig. 3 tells that, for both methods, details in the vegetation area and urban area are well-preserved, while the speckle noise in the water area is largely removed. However, Fig. 4 indicates that the proposed method achieves smoother regions than Pretest in the rural areas. In order to show more distinctions, Figs. 1 and 2 show one zoom-in area in each image. Both zoom-in images suggest that QMCTLS achieves smoother result than Pretest at the homogeneous areas.

Two statistics, equivalent number of looks (ENL) and edge-preservation degree based on ratio of average (EPD-ROA) [13] are used for measuring the despeckling performance. Larger ENL denotes better noise removal, while larger EPD-ROA means better detail preservation. The results are shown in Table I. The columns are the numerical measures by either ENL or EPD-ROA in the test regions corresponding to those in the red rectangles in Fig. 3 and Fig. 4. The EPD-ROA measurement values are averaged using 10 sample regions. The numerical measures are basically consistent with the visual observation. We notice that the Lee filter achieved slightly lower ENL values, but higher EPD-ROA values than Boxcar and IDAN methods. Overall, the ENL and EPD-ROA achieved by QMCTLS on both images are higher than the classical methods and the Pretest method. Particularly, QMCTLS achieves about twice the ENL value, and 27 percent higher EPD-ROA value than the average performances of classical methods on two test images. Compared with the Pretest method, QMCTLS achieves higher ENL and EPD-ROA values within the 1% significance level by the Kruskal-Wallis tests [14] for both images, indicating that with higher capability at detail preservation, the proposed QMCTLS method is also better at noise removal.

TABLE I

QUANTITATIVE COMPARISON BETWEEN TESTED METHODS. THE ENL-1 AND ENL-3 COLUMN REPRESENTS THE ESTIMATED ENL VALUES FOR REGION 1 AND REGION 3 RESPECTIVELY, AND THE EPD-ROA-2 AND EPD-ROA-4 COLUMN REPRESENTS THE AVERAGED ESTIMATED EPD-ROA VALUES FOR THE REGIONS LABELED AS 2 AND 4 RESPECTIVELY.

Methods	ENL-1	EPD-ROA-2	ENL-3	EPD-ROA-4
Original	3.11	1.00	9.25	1.00
Boxcar	66.19	0.62	220.71	0.54
IDAN	45.55	0.63	226.18	0.55
Lee	26.19	0.71	102.73	0.81
Pretest	159.16	0.90	414.29	0.93
QMCTLS	190.02	0.94	558.92	0.97

IV. CONCLUSION

A novel polarmetric SAR despeckling technique based on QMCS and region-based probabilistic texture similarity likelihood was presented. The stochastic QMCS was used to estimate the posterior probability. Also, in order to utilize the textural information, a regional probabilistic similarity measure based on the complex Wishart distribution was used to calculate the acceptance probability in QMCS. Experiments on two benchmark quad-pol SAR images demonstrated that the proposed QMCTLS method can effectively remove speckle noise, and in the meantime preserve image details. Future work is to replace the uniformly rectangular image regions with irregular regions based on local image property.

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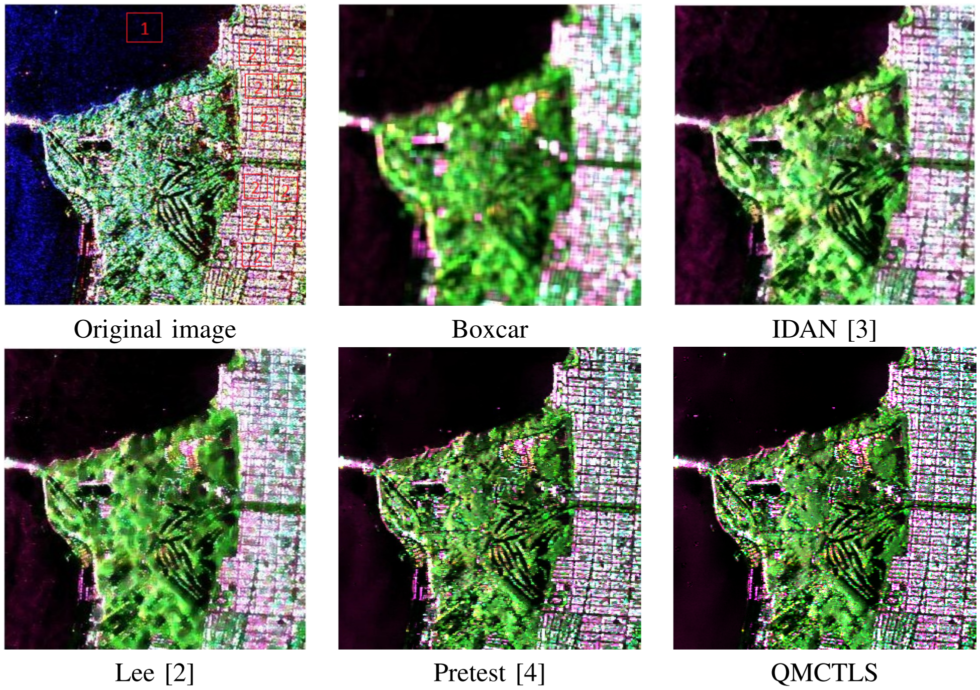

Fig. 3. Pauli-RGB of despeckling results on San Francisco image. Boxcar filter tends to blur the image. IDAN and Lee oversmooth the image to some degrees. The Pretest and the proposed QMCTLS filter preserve the details in the vegetation and urban areas very well, and, remove noise in the sea area.

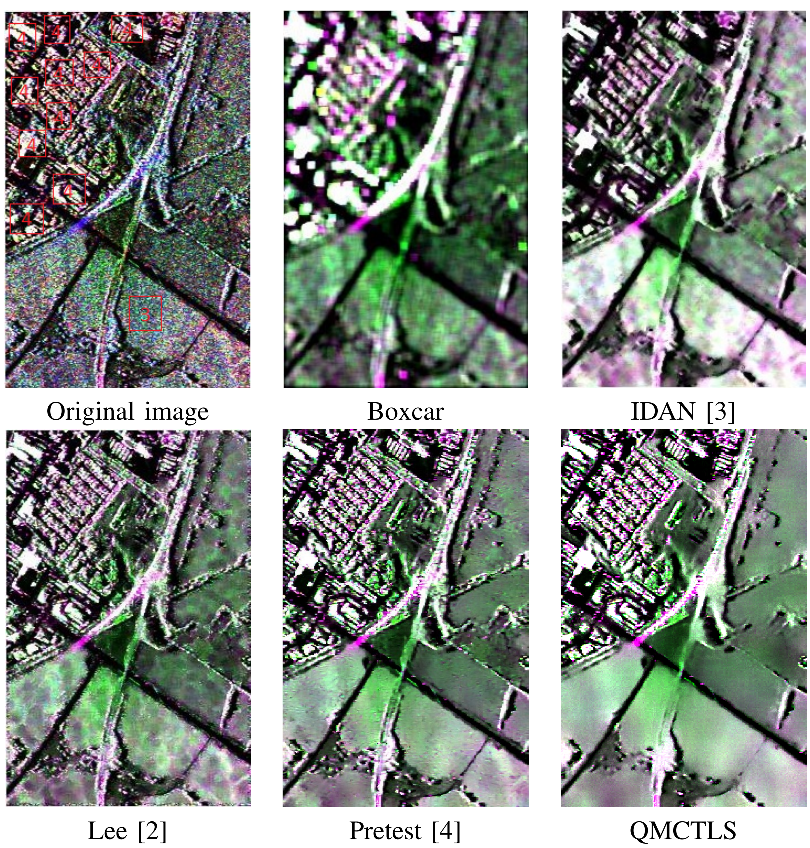

Fig. 4. Pauli-RGB of despeckling results for Ottawa image. Boxcar blurs the image greatly. IDAN is better, but still tends to lose details in the urban area. Lee filter seems to preserve undesirable artifacts in the farm area. Pretest and the proposed QMCTLS filter achieve better balance between noise removal and detail preservation, but QMCTLS achieves smoother results in farm area.