Hierarchical Methods for Global-Scale Estimation Problems

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Abstract

There is a substantial signal-processing challenge associated with large-scale (especially global-scale) remote-sensing problems: solving the statistical inverse problem (i.e., deducing the properties of the sensed field from measurements) by brute force, that is by covariance matrix inversion, is completely impractical for fields involving millions of pixels. This paper reports on the ongoing development of an alternative technique, in which the statistical problem is modeled on a multiscale tree, applied to estimating sea-surface temperature (SST) based on infrared radiance observations from the Along-Track Scanning Radiometers (ATSRs).

1 Introduction

The atmospheric and oceanographic sciences have recently experienced an explosion in the rate of remotely-sensed measurements, which has opened tremendous opportunities to study the ocean and associated climate at substantially improved time and space resolutions. Modern global climate models are running at resolutions as fine as one-twelfth degree, corresponding to global fields of 2000x4000 pixels, and remotely sensed observations, particularly of SST, are available at comparable resolution. The statistical manipulation and estimation of such large-scale models is the signal-processing challenge here: solving problems by brute force (matrix inversion) is completely impractical for fields involving millions of pixels.

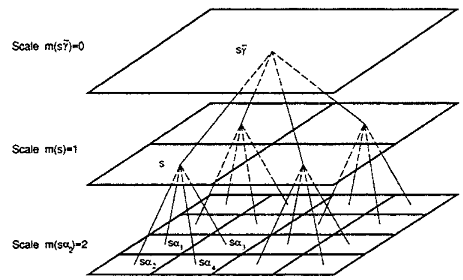

Figure 1: The standard quad-tree hierarchical structure on which a statistical model is built.

A number of methods have been proposed to address such large problems; FFT and multigrid techniques have both been used, however these make strong stationarity assumptions (FFT) or do not readily produce error statistics (multigrid). An alternative

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technique[2] has been developed which uses a hierarchical statistical structure on a tree (Figure 1). The approach is essentially one of divide and conquer: at each node on the tree, statistics are sought to conditionally decorrelate the subtrees extending from that node, consequently allowing each of the subtrees to be processed independently. In principle we are solving a classic estimation problem – widely known in the signal-processing community as the Kalman filter. This hierarchical model is already relatively mature, having been applied to oceanographic[3, 6], radar, medical, and groundwater[1] estimation problems; however the scale of recent global oceanographic problems presents substantial new challenges:

1. The problems are very large, in excess of one million pixels. At such dimensions it becomes difficult to satisfy the required conditional decorrelation, and numerical problems become prevalent.
2. The statistics of the prior model underlying the problem are nonstationary which complicates the parameterization of the problem.
3. Whereas previous applications of the multiscale framework have been to static problems, the nature of the SST estimation problem suggests assimilating data over time.

We will be illustrating our approach in the context of estimating the ocean-surface temperature (Figure 2), based on data from the ATSRs[7, 8] — a series of instruments mounted on the ERS-1/2 and ENVISAT research satellites. They use a unique scanning geometry to view the same point on the sea surface twice in rapid succession at two different angles through the intervening atmosphere, permitting SST to be retrieved with unprecedented accuracy. Global SST fields are generated at one-sixth degree resolution, but these are interrupted by clouds and by the width of the satellite swath, and

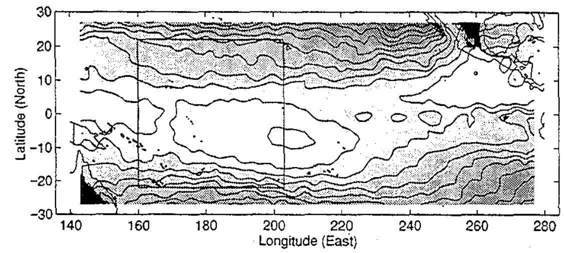

Figure 2: Problem context: 35-day mean sea-surface temperatures (one-degree contours) for April-May 1992 in the equatorial Pacific. The boxed region is analyzed in Section 3.

so need to be statistically interpolated to a regular grid for use in atmospheric or ocean modeling[9].

2 Multiscale Estimation

The multiscale statistical estimation method[2, 3] has, at its core, the following statistical model:

$$x(s) = A(s)x(s\bar{\gamma}) + B(s)w(s) \qquad (1)$$

where s is an index on a tree with parent $s\bar{\gamma}$, A and B are deterministic matrices, and w is a white-noise process. This equation essentially states a conditional decorrelation principle: the whiteness of w implies that the state $x(s\bar{\gamma})$ must conditionally decorrelate all child states connected to $s\bar{\gamma}$. The significance is that given a model in the form of (1) and a linear measurement model

$$y(s) = C(s)x(s) + v(s) \qquad (2)$$

then a *very* efficient, scale-recursive estimation algorithm is known to exist[2].

3 Application to SST

Figure 3 shows the state-vector definition which we will use: the state vector at a node

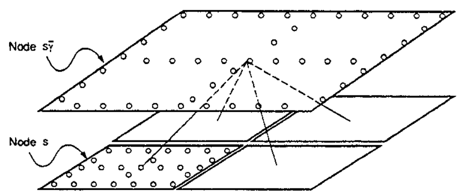

Figure 3: The definition of the state vectors: each node samples the boundaries of its four child-nodes.

s is made up of samples of the boundaries of its children, where the samples are spaced proportionately to the correlation length of the statistical process being estimated. For example, if the process being estimated were a first-order Markov random field, then sampling *every* pixel along the boundaries of the children of s would perfectly decorrelate the random fields of the children, thus perfectly satisfying (1). In practice we prefer to approximately satisfy (1), leading to suboptimal solutions but at vastly reduced computational effort.

Figure 4 shows a 256x256 array of measurements on the finest scale multiscale tree. The resulting estimates and error statistics, computed in about 15 seconds on a Sun Ultra-SPARC, are shown in Figures 5 and 6. The estimated field is smooth and relatively free of artifacts; the large-scale structure of the error variance field is correctly dependent upon the measurement distribution, although some the multiscale model introduces some patterning.

4 Extensions

The preceding section illustrated the estimation of large static random fields. The computational efficiency of the approach is motivating further research along two avenues: the estimation of much larger or higher resolution fields, and the dynamic estimation of processes over time.

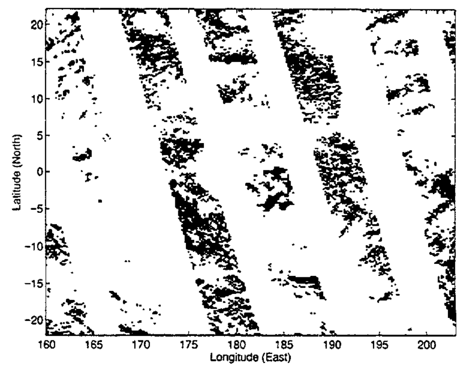

Figure 4: Spatial distribution of ATSR measurements, for a 3-day period using night-time observations only, in the central Pacific. Note the sampling irregularity due to clouds.

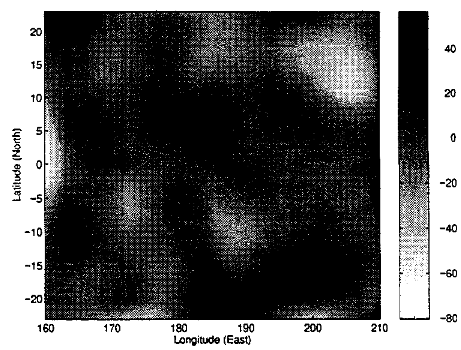

Figure 5: Estimates of SST anomalies (units in cK) about a 35-day mean based on the model of Figure 3, using 3 samples per correlation length on the node boundaries.

The estimation of very high-resolution fields is not a computational issue but a numeric one: as the state dimension is increased, at some point the matrix operations become unstable, illustrated in Figure 7. A square-root alternative to the estimation algorithm represents a possible solution, however we should not seek arbitrary increases in state dimension: a few measurements do not justify a plethora of finely-spaced state elements.

Dynamic estimation[4], *much* more com-

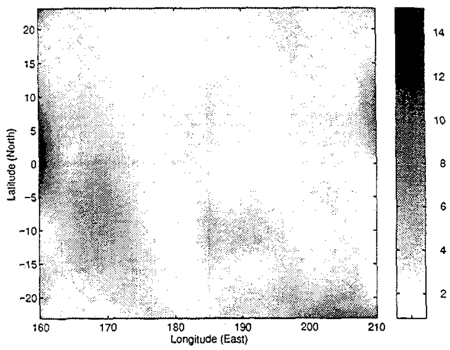

Figure 6: Estimation error std. dev. corresponding to the estimates of Figure 5. Our statistical model introduces some background patterning, which can be attenuated via finer sampling or reduced prior variances (eg, from improved mean fields).

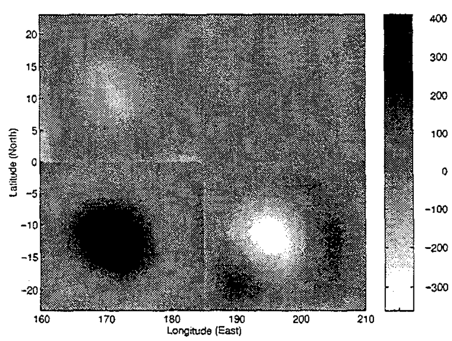

Figure 7: Estimates from a state model having 3.8 samples per correlation length, leading to numerical instabilities.

plicated than its static counterpart, is highly desirable in that it allows measurements distributed over time to contribute to estimates at some time instant without assuming that the field being estimated is static over time. We will start by using the estimate from one 3-day period as the prior mean for the next, yielding a simple causal filter. The next step is to incorporate some prior knowledge of ocean dynamics to improve on this simple "persistence" model, but this is the subject of future research.

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