Introduction

- Practical CRFs typically have edges only between nearby nodes.
- Using more interactions and expressive relations among nodes makes these methods impractical for large-scale applications (Computational Complexity).
- Fully connected CRFs can be tractable by defining specific potential functions [1].
- Inspired by random graph theory and sampling methods, a new clique structure called Stochastically Fully Connected Conditional Random Fields (SFCRFs) is proposed.
- The SFCRF is a marriage between random graphs and random fields.

Methodology

Stochastically fully connected conditional random fields (SFCRFs) are fully connected random fields in which cliques are defined stochastically:

\[ P(Y | X) = \frac{1}{Z(X)} \exp \left( -\psi(Y | X) \right) \]

where \( Z(X) \) is the partition function and \( \psi(\cdot) \) is some combination of unary and pairwise potential functions:

\[ \psi(Y | X) = \sum_{y_i \in Y} \psi_i(y_i; X) + \sum_{y_i, y_j \in X} \psi_{ij}(y_i, y_j; X) \]

Here \( y_i \in Y \) is a single state in the state set \( Y = \{ y_i \}_{i=1}^n \), \( Y \) is the set of states (clique), and \( X = \{ y_{ij} \}_{ij} \) is the set of observations.

Each node has a set of neighbors

\[ N(i) = \{ j | j = 1: n, j \neq i \} \]

where \( |N(i)| = n - 1 \).

The specified clique structure \( C \) is, in this paper, taken to be the pairwise clique

\[ C = \left\{ c_{ij}(i), j = 1 \right\} \]

\[ c_{ij}(i) = \{ (i, j) | j \in N(i), 1 \leq j \leq n, 1 \leq i \} \]

\[ 1_{ij}(c_{ij}) = \begin{cases} 1 & P_{ij} Q_{ij} \geq \gamma \\ 0 & \text{otherwise} \end{cases} \]

The threshold \( \gamma \) determines the sparsity of the graph.

Figure 1: A realization of a stochastically fully connected conditional random field graph. A connectivity between two nodes is determined based on a distribution; each two nodes in the graph can be connected according to a probability drawn from this distribution. There is a measurement \( x_i \) corresponding to each node \( y_i \). The connectivity of each pair of two nodes \( y_i \) and \( y_j \) is distinguished by the edge \( e_{ij} \)..Close nodes are connected with a higher probability (black solid edges), whereas two nodes with a greater separation are less probable to be connected (red dashed edges).

- The edges in \( G(\cdot) \) are randomly sampled, thus \( G \) is a realization of a random graph [2].
- If the probability \( p(\cdot) \) of the random graph \( G_{i, p} \) is greater than \( \frac{2n^2}{ \log(n) } \), the graph is connected with a high probability.
- In the experiments \( \gamma = 0.9993 \), meaning that the selection probability is equal to \( 7 \times 10^{-4} \).
- The underlying graph is connected since the selection probability is greater than \( \frac{2n^2}{ \log(n) } \), where \( n = 480 \times 360 \).
- An expected number of pairwise cliques is \( 2.09 \times 10^7 \) which is much smaller than \( n^2 \) that is \( 2.99 \times 10^{10} \) pairwise cliques in a fully connected graph.

Result

Table 1: Quantitative results (F1-score) based on the EnglishHnd dataset. The proposed framework is examined by two noise types with two different levels. The SFCRF is compared with the regular CRF (CRF-N3) and a CRF with a neighborhood size of 11 (CRF-N11). The per-iteration run time of each method is reported; all methods were run with an equal number of iterations.

<table>
<thead>
<tr>
<th>Observation Type</th>
<th>CRF-N3 F1-score</th>
<th>CRF-N11 F1-score</th>
<th>SFCRF F1-score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salt &amp; Pepper (80%)</td>
<td>0.488</td>
<td>0.872</td>
<td>0.911</td>
</tr>
<tr>
<td>Salt &amp; Pepper (90%)</td>
<td>0.235</td>
<td>0.313</td>
<td>0.859</td>
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<tr>
<td>Gaussian (20%)</td>
<td>0.566</td>
<td>0.818</td>
<td>0.895</td>
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<tr>
<td>Gaussian (30%)</td>
<td>0.391</td>
<td>0.646</td>
<td>0.842</td>
</tr>
<tr>
<td>Time per Iteration (s)</td>
<td>0.04</td>
<td>0.85</td>
<td>2.70</td>
</tr>
</tbody>
</table>

Figure 2: Qualitative results of SFCRF; the proposed algorithm is examined based on two noise types with two strengths. The results clearly show how the SFCRF outperforms both local and non-local CRFs.

Figure 3: The effectiveness of the SFCRF in the segmentation of noisy images from the Weizmann dataset. The first row shows the true images, the second the images after the distorted and the last row shows the segmentation results of the CRF-N3, CRF-N11 and the SFCRF respectively. The images are distorted with salt & pepper noise at 90%. The corresponding F1-score for each result is shown after the image. The results clearly illustrate the applicability of the SFCRF to natural images.

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References