EFFICIENT BAYESIAN INFERENCE USING FULLY CONNECTED CONDITIONAL RANDOM FIELDS WITH STOCHASTIC CLIQUES

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Introduction

- Practical CRFs typically have edges only between nearby nodes.
- Using more interactions and expressive relations among nodes make these methods impractical for large-scale applications (Computational Complexity)
- Fully connected CRFs can be tractable by defining specific potential functions [1]
- Inspired by random graph theory and sampling methods a new clique structure called stochastic cliques is proposed.
- The stochastically fully connected CRF (SFCRF) is a marriage between random graphs and random fields.

Methodology

Stochastically fully connected conditional random fields (SFCRFs) are fully connected random fields in which cliques are defined stochastically:

$$P(Y|X) = \frac{1}{Z(X)}\exp(-\psi(Y|X)) \tag{1}$$

where $Z(X)$ is the partition function and $\psi(.)$ is some combination of unary and pairwise potential functions:

$$\psi(Y|X) = \sum_{i=1}^{n}\psi_u(y_i,X) + \sum_{\varphi\in C}\psi_p(y_\varphi,X) \tag{2}$$

Here $y_i \in Y$ is a single state in the set $Y=\{y_i\}_{i=1}^n$, $y_\varphi \in Y$ is the subset of states (clique), and $X=\{x_j\}_{j=1}^n$ is the set of observations.
Each node i has a set of neighbors

$$N(i) = \Big\{ j | j = 1:n, j \neq i \Big\} \tag{3}$$

where $|N(i)| = n-1$.
The specified clique structure C is, in this paper, taken to be the pairwise clique

$$C = \Big\{ C_p(i) \Big\}_{i=1}^{n} \tag{4}$$

$$C_p(i) = \Big\{ (i,j) | j \in N(i), 1^S_{\{i,j\}} = 1 \Big\} \tag{5}$$

$$1^S_{\{i,j\}} = \begin{cases} 1 & P^s_{i,j}\cdot Q^d_{i,j} \geq \gamma \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

$1^S_{\{i,j\}} = 1$ has two responsibilities:

1. Incorporate the spatial information ($P^s_{i,j}$)
   The connectedness probability of two nodes and the distance between them are inversely related.
2. Utilize the data relation information among the states ($Q^d_{i,j}$)
   The connectedness probability of two nodes is directly related to the color similarity between them.

The threshold γ determines the sparsity of the graph.

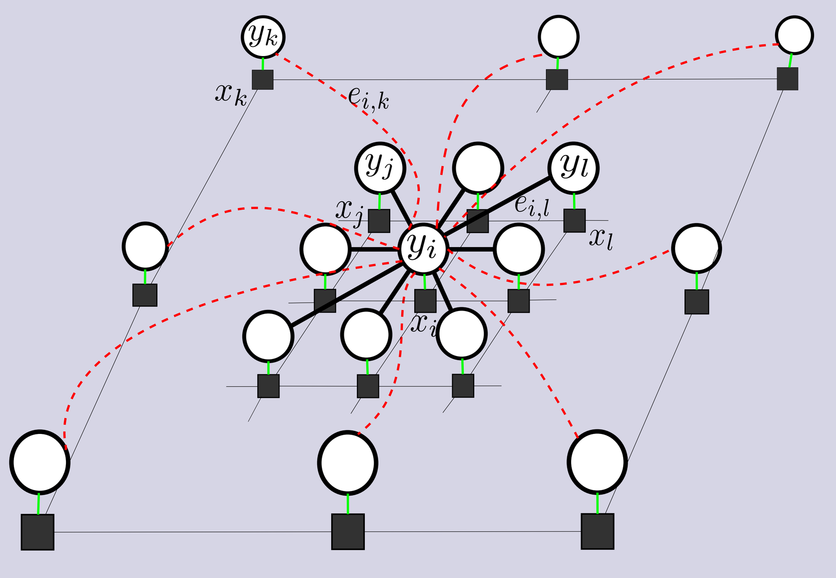

Figure 1: A realization of a stochastically fully connected conditional random field graph. A connectivity between two nodes is determined based on a distribution; each two nodes in the graph can be connected according to a probability drawn from this distribution. There is a measurement x_i corresponding to each node y_i. The connectivity of each pair of two nodes y_i and y_k is distinguished by the edge $e_{i,k}$. Closer nodes are connected with a higher probability (black solid edges), whereas two nodes with a greater separation are less probable to be connected (red dashed edges).

- The edges in $G(.)$ are randomly sampled, thus G is a realization of a random graph [2].
- If the probability p' of the random graph $\hat{G}_{n,p'}$ is greater than $\frac{\log n}{n}$ the graph is connected with a high probability.
- In the experiments $\gamma = 0.9993$, meaning that the selection probability is equal to 7×10^{-4}.
- The underlying graph is connected since the selection probability is greater than $\frac{\log n}{n}$, where $n = 480\times360$.
- An expected number of pairwise cliques is 2.09×10^7 which is much smaller than n^2 that is 2.99×10^{10} pairwise cliques in a fully connected graph.

Result

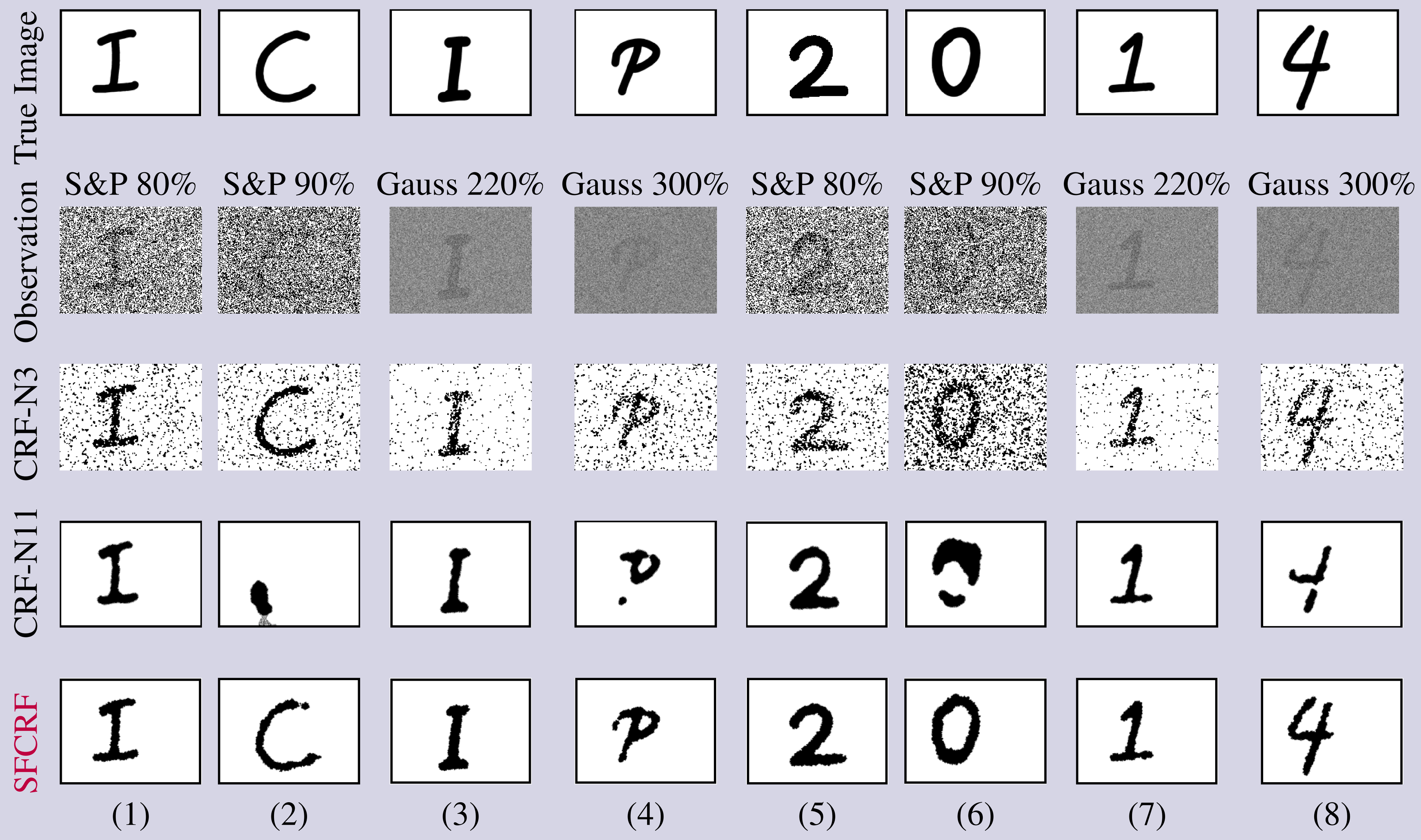

Figure 2: Qualitative results of SFCRF; the proposed algorithm is examined based on two noise types with two strengths. The results clearly show how the SFCRF outperforms both local and non-local CRFs.

Table 1: Quantitative results (F1-score) based on the EnglishHnd dataset. The proposed framework is examined by two noise types with two different levels. The SFCRF is compared with the regular CRF (CRF-N3) and a CRF with a neighborhood size of 11 (CRF-N11). The per-iteration run time of each method is reported; all methods were run with an equal number of iterations.

	CRF-N3	CRF-N11	SFCRF
Salt & Pepper (80%)	0.488	0.872	0.931
Salt & Pepper (90%)	0.235	0.313	0.859
Gaussian (220%)	0.566	0.818	0.895
Gaussian (300%)	0.391	0.646	0.842
Time per Iteration (s)	0.04	3.85	2.70

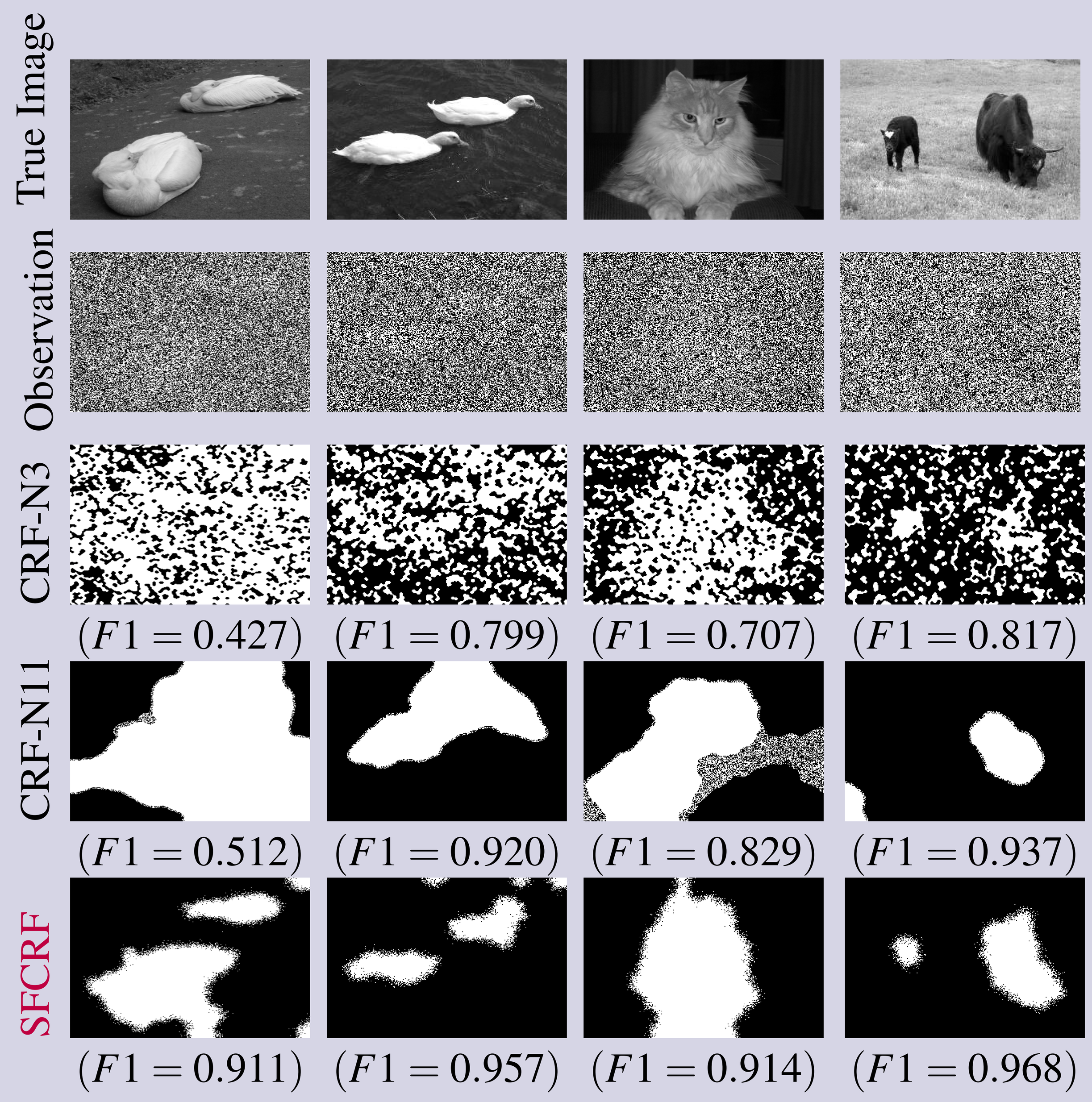

Figure 3: The effectiveness of the SFCRF in the segmentation of noisy images from the Weizmann dataset. The first row shows the true images, the second the images after distortion, and the three last rows the segmentation results of the CRF-N3, CRF-N11 and the SFCRF respectively. The images are distorted with salt & pepper noise at 90%. The corresponding F1-score for each result is shown after the image. The results clearly illustrate the applicability of the SFCRF to natural images.

Acknowledgement

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References

[1] P. Krähenbühl and V. Koltun, "Efficient inference in fully connected CRFs with Gaussian edge potentials," in *Advances in Neural Information Processing Systems*, 2011.

[2] P. Erdos and A. Renyi, "On random graphs i.," *Publ. Math. Debrecen*, 1959.