Image Resolution Enhancement with Hierarchical Hidden Fields

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Abstract. In any image processing involving images having scale-dependent structure, a key challenge is the modeling of these multi-scale characteristics. Because single Gauss-Markov models are effective at representing only single-scale phenomena, the classic Hidden Markov Model can not perform well in the processing of more complex images, particularly near-fractal images which frequently occur in scientific imaging. Of further interest is the presence of space-variable, nonstationary behaviour. By constructing hierarchical hidden fields, which label the behaviour type, we are able to capture heterogeneous structure in a scale-dependent way. We will illustrate the approach with a method of frozen-state simulated annealing and will apply it to the resolution enhancement of porous media images.

1 Introduction

There are many problems in texture analysis, remote sensing and scientific imaging where we observe scale-dependent structure. We are interested in the reconstruction or enhancement of multi-scale structures on the basis of low-resolution measurements.

Hidden Markov Fields (HMFs) are widely used in image restoration and resolution enhancement [1,2,3], however because Markov Random Field (MRF) models (and most other local models) can only describe structures at a single scale, most methods fail to produce convincing fine-scale and coarse-scale reconstructions. Recently, parallel MRFs [4] based on Triplet MRFs [5] allow representing multi-scale structures by modeling on multiple, parallel random fields, however in practice this method is unwieldy for modeling anything beyond only a few scales.

On the other hand, creating a prior based on a hierarchical structure provides a more natural way to introduce scale-dependent models. Alexander and Fieguth [6] proposed such a hierarchical model, achieved by specifying a set of single MRFs to capture the features at each scale, however this model ignores the interrelation between scales. Kato et al [7] introduced a 3D neighborhood system, but at considerable computational cost. Later, Mignotte et al [8] proposed a relatively simply spatial interrelation defined by a Markov chain from parent

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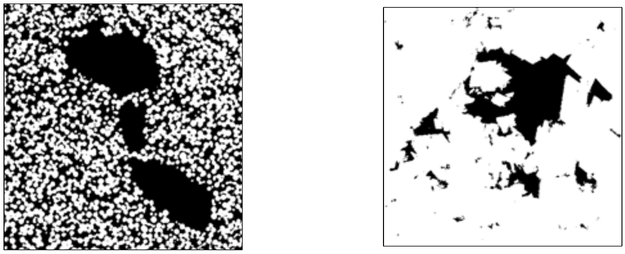


(a) Sintered glass beads (b) Carbonate rock

Fig. 1. Excerpts of two large microscopic images of physical porous media. The scale-dependent pore structure (left) and the near-fractal behaviour (right) is clearly evident and poses challenges for image reconstruction.

to children, however the model at each scale is too simple to allow significant structure.

Most recently, Campaigne et al [9] proposed a frozen-state hierarchical annealing method, which attractive computational complexity and good scale-dependent modeling. In [9] the method was only used for random synthesis; in this paper we wish to modify the method, together with a hierarchical hidden field, to perform resolution enhancement. We will apply our methods to scientific images from porous media, such as those in Fig. 1, since they include complex, fractal-like structures which are very challenging in resolution enhancement.

2 Hidden Markov Field

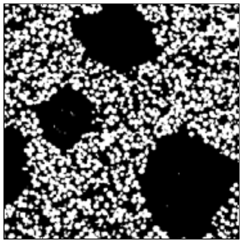
A classical HMF [1] has two layers: an observable field (Y) and a hidden field (X) to be estimated. Let us define $Y = \{Y_s : s \in S_L\}$ where S_L is a LR grid space and $X = \{X_s : s \in S_H\}$ where S_H is a HR grid space. Therefore, we have

$$Y = g(X) + \nu \tag{1}$$

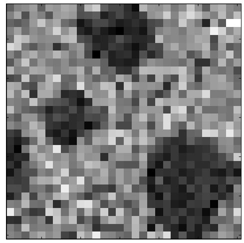
where $g(\cdot)$ denotes the forward operation, and ν denotes measurement noise. A resolution enhancement problem is to generate an estimate $\hat{x}$ from y:

$$p(x|y) = \prod_{s \in S_L} p(y_s|x)p(x) \tag{2}$$

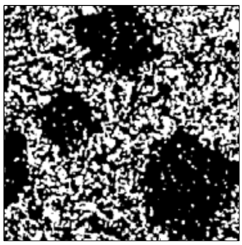
In this classical HMF, the prior of the ideal HR field X is assumed to be MRF. However, a single local MRF can not work well in modeling multi-scale structures where X is generally nonstationary (Fig. 2(a)). For example, for a noisy image with two-scale structure (Fig. 2(b)), the classical HMF method fails to reconstruct the large scale pores (Fig. 2(c)).



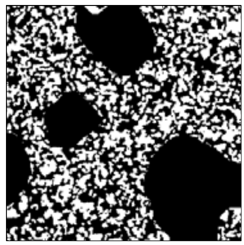
(a) HR ground truth $\overset{*}{x}$



(b) LR, noisy measurement y



(c) HR estimate $\hat{x}$ from the classical HMF [3]



(d) HR estimate $\hat{x}$ from the Multiple Parallel MRFs [4]

Fig. 2. Suppose we have a two-scale structure (a), here a set of glass beads. A single hidden Markov model is poor at simultaneously modeling the large black pores and tiny spaces between beads, therefore the HR result (c) using a classical HMF model can not strongly assert the presence of large scale structures. Instead, (d) shows that multiple (parallel) MRFs leads to an improved result with this two-scale case, however the parallel method is tractable only for problems having a limited number of scales.

3 Hierarchical Markov Field

A Hierarchical Markov field can define the ideal HR field X as series of random fields $\{X^k,\, k \in K = (0, 1, \cdots, M)\}$ in a hierarchical structure, where $k = 0$ defines the finest scale. At each scale k, X^k is defined on site space S^k and is resulted from downsampling X^0 by $2^k \times 2^k$. The whole site space of the hierarchy can be defined as $S = \cup_{k=0}^{M} S^k = \{s_1, s_2, \cdots, s_N\}$.

According to Mignotte et al [8], the spatial contextual interrelation is defined by two random processes:

1. An inter-layer causal Markov chain: $p(x^k|x^{K\setminus k}) = p(x^k|x^{k+1})$
2. An intra-layer Markov Field: $p(x_s^k|x_{S\setminus s}^k) = p(x_s^k|x_{\wp(s)}^{k+1}, x_{\mathcal{N}_s^k}^k)$

Here, if s at scale k, $\wp(s)$ denotes the parent site of s at scale $(k+1)$, and $\mathcal{N}_s^k$ defines a the local neighborhood of s at the same scale as s.

As this model [8] is still computationally expensive, to achieve faster convergence Campaigne et al [9] proposed a hierarchical model with a frozen-state. In

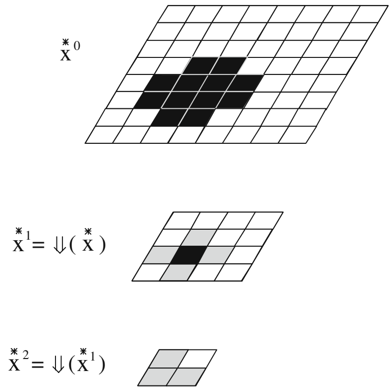


Fig. 3. An example of frozen-state hierarchical structure [9]: a given field $\overset{*}{x}^0$ is coarsified by repeated 2×2 subsampling $\Downarrow(\cdot)$. All-white and all-black regions are preserved, with mixtures labelled as uncertain (grey).

their work, a HR field ($\overset{*}{x} = \overset{*}{x}^0$) can be represented by a hierarchical field $\{\overset{*}{x}^k\}$ (Fig. 3) where $\overset{*}{x}^k = \Downarrow^k (\overset{*}{x}^0)$, and $\Downarrow^k (\cdot)$ denotes a downsampling operator. At each scale, only those sites which are undetermined need to be sampled, with the remainder fixed (or frozen). The site sampling strategy is

$$x_s^k = \begin{cases} x_s^{k+1} & \text{with } p(x_s^k | x_s^{k+1}) = 1, \text{ if } x_s^{k+1} \in \{0, 1, \cdots, n\} \\ \text{a sample from } p(x_s^k | x_s^{k+1}, x_{\mathcal{N}_s^k}^k), & \text{if } x_s^{k+1} = \frac{1}{2} \end{cases} \tag{3}$$

where $\frac{1}{2}$ denotes an undetermined state.

4 Hierarchical Hidden Fields

4.1 Single Hierarchical Hidden Field

Under the HMF framework, if we use a hierarchical field to model X, a single Hierarchical Hidden Markov Field (HHMF) model can be written as

$$p(x|y) \propto \prod_{s \in S_L} p(y_s|x) \cdot \prod_{k=0}^{M-1} p(x^k|x^{k+1}) \cdot p(x^M) \tag{4}$$

where $k = M$ denotes the coarsest scale of X.

In (4), if we apply a frozen-state hierarchical model to define $\{p(x^k|x^{k+1})\}$, at each scale the model only needs to capture the characteristics of the sites whose states have not be determined at the parent scale. However, in the absence of additional prior knowledge, this single hierarchy will contain a stationary prior model, whereas many random fields have some sort of nonstationary piece-wise multi-model behaviour, which requires an additional hidden field.

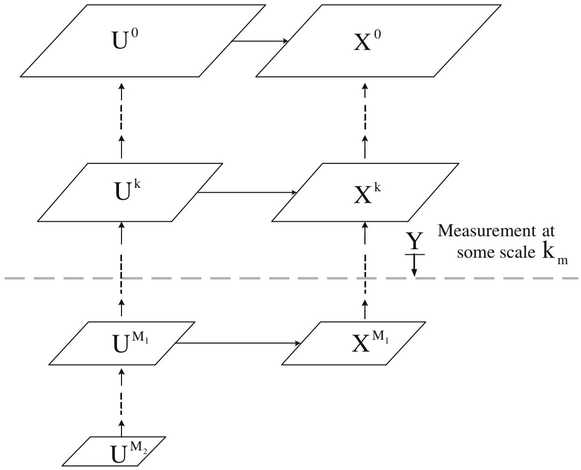


Fig. 4. The proposed Pairwise Hierarchical Hidden Markov Fields modeling structure, such that the hidden field U represents a label or description of the scale-dependent behavior of X. Coarser than some scale M_1 the visible field is all-grey, and so only the hidden field is represented to scale M_2.

4.2 Pairwise Hierarchical Hidden Field

To further generalize the modeling of $\{X^k\}$ we develop a new Pairwise Hierarchical Markov Field (PHMF) $\{(X^k, U^k),\ k \in K = (0, 1, \cdots, M)\}$. Here U is introduced as an auxiliary hierarchical field $\{U^k\}$ to capture the label of large scale nonstationarities of $\{X^k\}$. Now, what we need is a series of pairwise fields $\{(X^k, U^k)\}$, rather than $\{X^k\}$, to be Markov, so the assumption on X is relaxed. Since for a enhancement process we know $p(y|x^k, u^k) = p(y|x^k)$, the PHHMF model, illustrated in Fig. 4, can be written as

$$p(x, u|y) \propto \prod_{s \in S_L} p(y_s|x) \cdot \prod_{k=0}^{M_1-1} p(x^k|x^{k+1}, u^k) \cdot p(x^{M_1}|u^{M_1}) \cdot \prod_{k=M_1}^{M_2-1} p(u^k|u^{k+1}) \cdot p(u^{M_2}) \tag{5}$$

where $k = M_1$ and $k = M_2$ denote the coarsest scale of X and U respectively. Since $\{U^k\}$ is defined to describe large scale features or model behaviour in X, the decidable state in X is expected to vanish at a finer scale than in U, so we assume $0 \leq k_m \leq M_1 \leq M_2$, where k_m denotes the measurement scale.

Given measurements Y contaminated by i.i.d. noise, the posterior distribution of (X, U, Y) can be represented as a Gibbs distribution

$$p(x, u|y) = \frac{e^{-\frac{1}{T}E(x,u|y)}}{\mathcal{Z}} \tag{6}$$

where $\mathcal{Z}$ is the partition function, T is the temperature, and E is the energy function describing the relationships among Y, X and U, capturing the interactions among the state elements in the random fields.

As $p(x,u|y)$ is defined by PHHMFs and $E(x,u|y)$ can be specified as

$$E(x,u|y) = E_m(y|x) + \sum_{k=0}^{M_1-1} E^k_{x|u}(x^k|x^{k+1},u^k) + E^{M_1}_{x|u}(x^{M_1}|u^{M_1}) + \sum_{k=M_1}^{M_2-1} E^k_u(u^k|u^{k+1}) + E^{M_2}_u(u^{M_2}) \quad (7)$$

where $\{E^k_{x|u}\}$ and $\{E^k_u\}$ are prior models and $E_m(\cdot)$ is the corresponding measurement energy. We assume $E_m(\cdot)$ to be Gaussian, whereas the priors are learned using a nonparametric joint local distribution [6,9] from downsampled training data $\bar{x}^k = \Downarrow^k (\bar{x}^0|\bar{u}^0)$ and $\bar{u}^k = \Downarrow^k (\bar{u}^0)$ respectively.

Given the posterior energy (7), we can generate samples $\hat{x}$ and $\hat{u}$. The algorithm here we use is a hierarchical frozen-state annealing processes [9], as outlined in Algorithm 1:

Algorithm 1. Pairwise Hierarchical Hidden Annealing

Function $(\hat{x},\hat{u}) = PHHA\,(\{E^k_m\},\{E^k_{x|u}\},\{E^k_u\})$
1: $\{\hat{u}^k, 0 \le k \le M_2\} = SHA\,(\{E^k_m\},\{E^k_u\})$
2: $\{\hat{x}^k, 0 \le k \le M_1\} = SHA\,(\{E^k_m\},\{E^k_{x|u}\})$

such that the hierarchy is sampled twice, once for the label U, and secondly for the field X, but both driven by the same measurement set Y. Each hierarchy is sampled over all scales:

Algorithm 2. Single Hierarchy Annealing

Function $\{\hat{x}^k\} = SHA\,(\{E^k_m\},\{E^k_x\})$
1: $\hat{x}^M \leftarrow$ sample with Simulated Annealing [1], M is the coarsest scale
2: **repeat** from $k = M-1$
3: $\hat{x}^k = FSA\,(\hat{x}^{k+1}, E^k_m, E^k_x)$
4: $k \leftarrow k-1$
5: $\hat{x}^k \leftarrow \Uparrow (\hat{x}^{k+1})$ map to the finer scale [9]
6: **until** $k \rightarrow 0$ the finest scale

The sampling at each scale, shown in Algorithm 3, proceeds via the frozen-state method (3).

The benefit of PHHMF is obvious by comparing with existing models. First, it reduces the modeling approximation. The proposed model is more satisfied with the Markov assumption required by the HMF framework. Second, it enhances the modeling capability considerably but maintaining computational tractability.

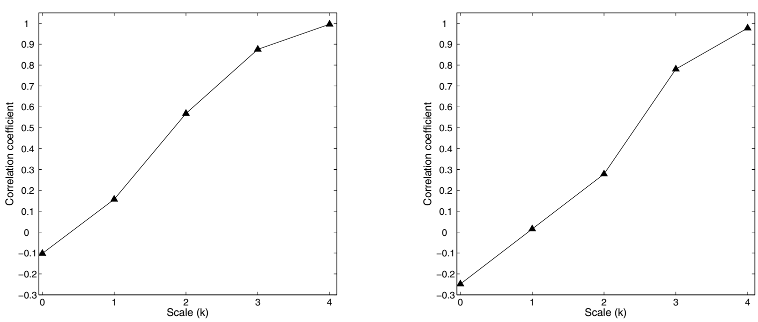
Algorithm 3. [1] [9] Frozen-State Annealing

Function $x^k = FSA\,(x^{k+1}, E_m, E_x)$
for $s \in S$ **do**
  if $(x_s^{k+1} = \frac{1}{2})$
    $i \leftarrow 0$
    repeat
      $\beta_i = 1/T^{(i)}$
      $x_s^{k(i)} \leftarrow$ sample $p^{\beta_i}(x|y) \propto \frac{1}{Z} exp\{-\beta_i(E_m + E_x)\}$ given $x^{(i-1)}$
      $i \leftarrow i + 1$
    until $T \rightarrow 0$
  else
    $x_s^k \leftarrow x_s^{k+1}$
  end if
end for

5 Results and Evaluation

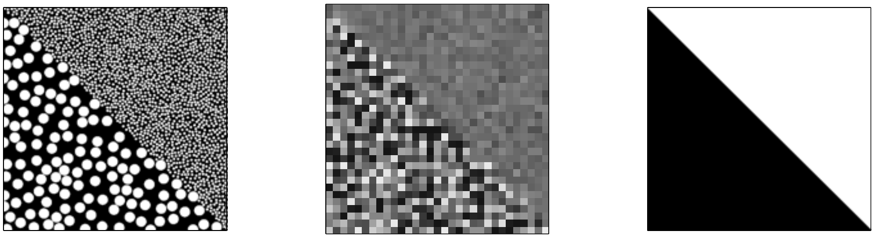
To test the resolution enhancement approach based on our proposed model we use three data sets: a synthetic small-large circle example, and two real images of porous materials.

In small-large circle example, we assume the field U, which labels the non-stationarity of the HR field, is known (Fig. 6(c)). Given a LR noisy image (Fig. 6(b)), our estimated results $\{\hat{x}^k, 0 \le k \le 4\}$ are shown in (Fig. 6(d)-(h)). Clearly, in the resolution enhancing process the structures of the two-scale circles are gradually decided from coarse to fine. We also notice that both small and large scale structures are separately reconstructed, using separate models, rather than forced to fit a single model (as in Fig. 6(i)), illustrating the positive effect of the U field to label the nonstationary behaviour.



(a) Sintered glass beads (b) Carbonate rock

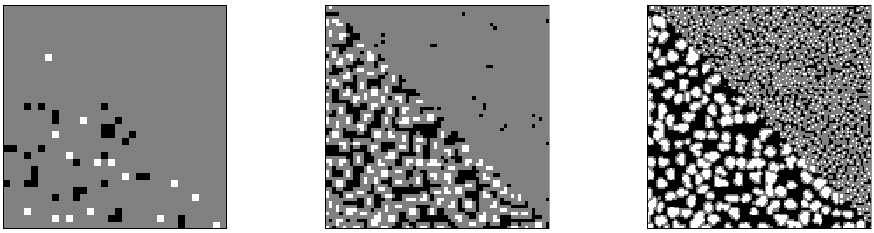
Fig. 5. Correlation coefficients ρ between the estimates $\hat{x}$ and ground truth $\overset{*}{x}$ as a function of scale. For a number of scales below the measured resolution k_m, $\rho(\overset{*}{x}, \hat{x}) > 0$ meaning that some trustable details are reconstructed.



(a) HR ground truth $\overset{*}{x}$, 512×512

(b) LR, noisy y, 32×32

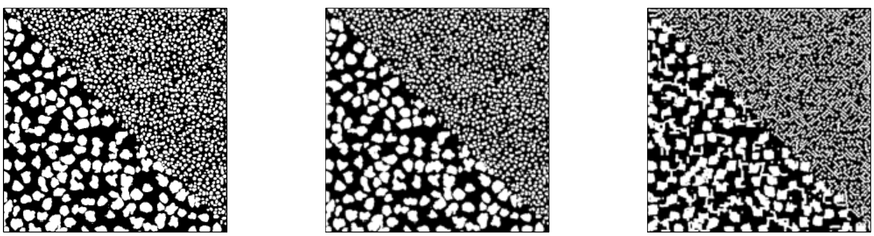
(c) Assumed hidden label field $\overset{*}{u}$, 512×512



(d) $\hat{x}^4$, 32×32

(e) $\hat{x}^3$, 64×64

(f) $\hat{x}^2$, 128×128



(g) $\hat{x}^1$, 256×256

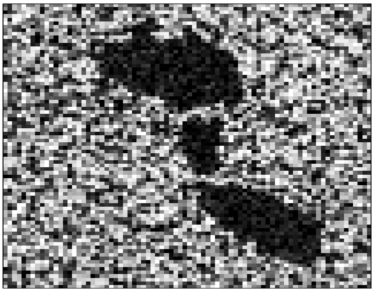
(h) HR estimate $\hat{x}$ from PHHMFs, 512×512

(i)HR estimate $\hat{x}$ from [10], 512×512

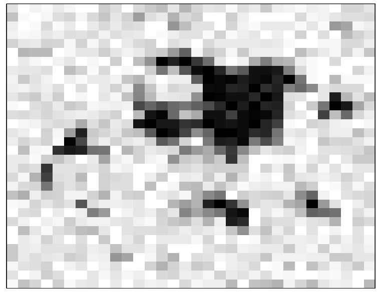
Fig. 6. A toy two-scale problem. For the purpose of this example, we assume the nonstationarity label field u to be given (c). From the low resolution measurements of (b), our estimated results $\{\hat{x}^k, 0 \leq k \leq 4\}$ are shown in (d)-(h). The clear scale separation of the result in (h) should be compared to the stationary, classic annealing result in (i) from [10].

In the porous media examples, we estimate both x and u hierarchically. The HR estimate ($\hat{x}$) for glass beans is shown in Fig. 7(c), and the $\hat{x}$ for carbonate rock porous media is shown in Fig. 7(d).

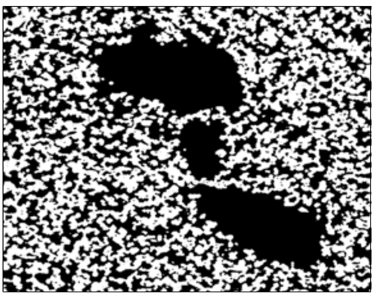
We evaluate $\hat{x}$ in respect of how much it is consistent with the HR ground truths ($\overset{*}{x}$) from which the y is obtained at scale $k = k_m$. The correlation coefficient $\rho(\overset{*}{x}, \hat{x})$ is studied as a function of scale. The $\rho(\overset{*}{x}, \hat{x})$ for glass bean and



(a)Measurement, $y = g(\overset{*}{x}) + \omega$, 80×80, $k_m = 4$

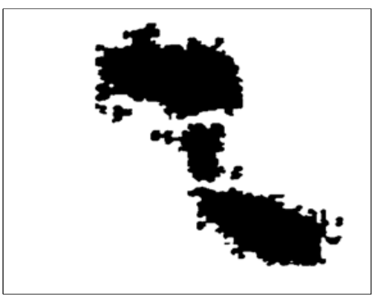


(b) Measurement, $y = g(\overset{*}{x}) + \omega$, 32×32, $k_m = 4$

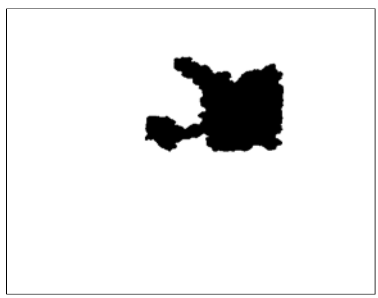


(c) HR estimate, $\hat{x}$, 640×640, $k = 0$

(d) HR estimate, $\hat{x}$, 512×512, $k = 0$



(e)Estimated label field,$\hat{u}$ 640×640, $k = 0$



(f) Estimated label field,$\hat{u}$ 512×512, $k = 0$

Fig. 7. Enhancement results for sintered glass bead image and carbonate rock image. In sintered glass bead case, observe the faithful reconstruction in $\hat{x}$ (c) of even subtle density fluctuations in the HR ground truth (Fig. 1(a)) (eg., top-right corner) and of subtle connectivities (eg., at the interface between the large pores). In carbonate rock case, the flat facets visible in the HR ground truth (Fig. 1(b)) are not able to be fully reproduced by the local model in $\hat{x}$ (d), nevertheless the improvement in relevant detail of (d) over (b) is stunning.

carbonate rock images are plotted in Fig. 5(a) and Fig. 5(b) respectively. As $\rho(\overset{*}{x}, \hat{x}) > 0$, it means some enhancement details can be trust.

During the enhancement process, without considering noise, the measurements y at scale k_m do provide information on structures at scales $k \leq k_m$. Clearly, as $k \to 0$, y provides fewer and fewer constraints on the details, so the correlations are expected to be decreasing at finer scales.

6 Conclusion

In this paper, a new Pairwise Hierarchical Hidden Markov Field model is proposed. Based on the proposed model, an resolution enhancement approach is set up to deal with images with multi-scale phenomena. By introducing an auxiliary hierarchical hidden field to gradually label the nonstationarity in the images, the our model shows impressive capability to captured the multi-scale statistical characteristics with maintaining computational tractability. Although here the proposed approach is only applied for porous media images, it can be extended to solve more general texture-related problems in modeling, analysis, and processing.

References

1. Geman, S., Geman, D.: Stochastic relaxation, gibbs distributions, and the bayesian restoration of images. IEEE Trans. PAMI 6(6), 721–741 (1984)
2. Torquato, S.: Random heterogeneous materials: microstructure and macroscopic properties. Springer, Heidelberg (2002)
3. Mohebi, A., Fieguth, P.: Posterior sampling of scientific images. In: Campilho, A., Kamel, M.S. (eds.) ICIAR 2006. LNCS, vol. 4141, pp. 339–350. Springer, Heidelberg (2006)
4. Liu, Y., Mohebi, A., Fieguth, P.: Modeling of multiscale porous media using multiple markov random fields. In: 4th Biot, June 2009 (accepted)
5. Benboudjema, D., Pieczynski, W.: Unsupervised statistical segmentation of nonstationary images using triplet markov fields. IEEE Trans. on PAMI 29(8), 1367–1378 (2007)
6. Alexander, S.K., Fieguth, P., Vrscay, E.R.: Hierarchical annealing for random image synthesis. In: Rangarajan, A., Figueiredo, M.A.T., Zerubia, J. (eds.) EMMCVPR 2003. LNCS, vol. 2683, Springer, Heidelberg (2003)
7. Kato, Z., Berthod, M., Zerubia, J.: A hierarchical markov random field model and multitemperature annealing for parallel image classificaion. Graphical Models and Image Proceesing 58(1), 18–37 (1996)
8. Mignotte, M., Collet, C., Pérez, P., Bouthemy, P.: Sonar image segmentation using an unsupervised hierarchical mrf model. IEEE Trans. Image Processing 9(7), 1216–1231 (2000)
9. Campaigne, W.R., Fieguth, P., Alexander, S.K.: Frozen-state hierarchical annealing. In: Campilho, A., Kamel, M.S. (eds.) ICIAR 2006. LNCS, vol. 4141, pp. 41–52. Springer, Heidelberg (2006)
10. Mohebi, A., Fieguth, P.: Statistical fusion and sampling of scientific images. In: ICIP 2008, pp. 1312–1315. IEEE, Los Alamitos (2008)