# Large Scale Dynamic Estimation Of Ocean Surface Temperature

P.W. FIEGUTH F.M. KHELLAH
University of Waterloo
Waterloo, Ont., N2L 3G1, Canada

M.J. MURRAY M.R. ALLEN
Rutherford Appleton Laboratory
Chilton, Didcot, OX11 OQX, UK

pfieguth@uwaterloo.ca Tel: (519) 888-4567 x3599 Fax: (519) 746-4791

## ABSTRACT

This paper addresses the dynamic estimation of the ocean surface temperature based on data from the Along-Track Scanning Radiometer (ATSR) for large ($512 \times 512$) fields. For such huge problems, the conventional dynamic estimation tool (the Kalman filter) is not directly applicable, instead, we develop a recursive estimation algorithm that *emulates* the Kalman filter. Our approach uses a recently-developed multiscale estimation algorithm for the update step, and makes simplifying assumptions about the surface dynamics leading to a computationally efficient prediction step.

## INTRODUCTION

Dynamic models, long the focus of global circulation and related climate models, have been slow to be adopted for very large statistical estimation problems. This is primarily due to the $\mathcal{O}(n^3)$ computational complexity of the Kalman filter for $n$ pixels, which is totally impractical for mega-pixel remotely-sensed fields.

For problems which are very densely measured, it may be adequate to employ a *static* optimal interpolation procedure, incorporating purely spatial statistics, which assumes that the underlying field is constant over time. However a dynamic approach has much to offer: the field is not assumed to be constant, a prior-mean field does not need to be defined, and meaningful estimates can be computed in the presence of data gaps.

In this paper, we discuss a method of dynamic estimation for sea surface temperature SST data from the along-track scanning radiometer (ATSR); for which a sample 3-day set of measurements is shown in Fig.1. The ATSR data is well-suited to a dynamic approach: the SST is time-varying but also highly-correlated over time, the SST does not really possess a well-defined mean field, and there are frequent (and sometimes widespread) data gaps due to clouds.

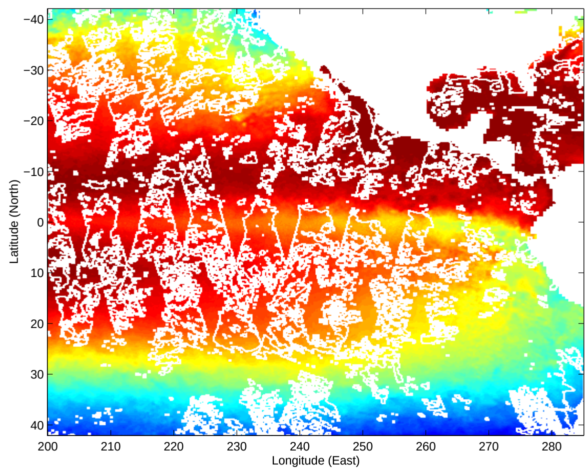


Figure 1: Superposition of ocean surface temperature measurements and the ocean yearly mean.

## DYNAMIC ESTIMATION

The dynamic evolution of SST is assumed to obey the linear discrete dynamic model given by:

$$x(t+1) = Ax(t) + Bw(t) \tag{1}$$

where $w(t) \sim \mathcal{N}(0, Q)$ is an uncorrelated Gaussian noise process, with zero mean, diagonal covariance $Q$.

The ATSR measurements are linearly related to SST:

$$y(t) = C(t)x(t) + v(t) \tag{2}$$

where $v(t) \sim \mathcal{N}(0, R)$ is an uncorrelated Gaussian noise process, zero mean, diagonal covariance $R$.

For physical systems governed by models (1), (2), the Kalman filter can be used to obtain filtered estimates for the state $x(t)$ at time $t$ based on the data available up to time $t$.

Alternatively, it may be preferable to use a smoothing filter, in which the state $x(t)$ is estimated acausally,

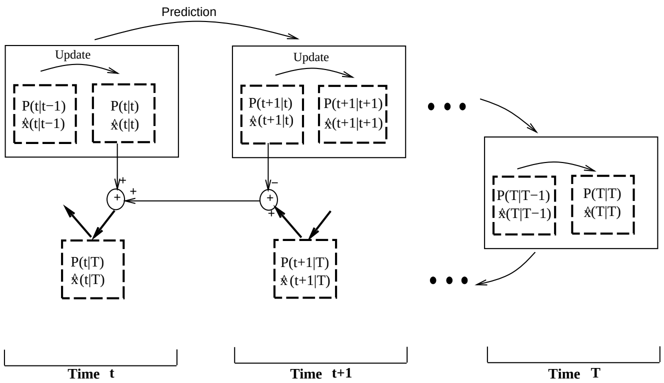


Figure 2: The basic principle of bi-directional filtering over time, consisting of the Kalman filter (forwards) and a smoothing filter (backwards).

based on data both preceding and following time $t$. A smoothing filter can lead to substantial reductions in estimation error, particularly when the measurements are sparse or noisy, and when the physical process is strongly correlated over time. Smoothed estimates are computed based on all the available data $0 \leq t \leq T$ in two passes, as shown in Fig. 2: a forward pass, identical to the Kalman filter, and a backward pass, initialized by the filtered estimates at end time $T$.

Our approach is to *emulate* the update and prediction steps of the Kalman filter without trying to solve them exactly (which is computationally prohibitive). The update step is static in nature, and we propose to use an existing, efficient technique based on a multiscale (hierarchical) framework [1, 4]. This technique can accommodate the irregular measurements and nonstationary prior models encountered in the SST problem; it attains computational and storage efficiency by constructing *models* for the needed statistics, rather than generating large covariance matrices. For two-dimensional fields a quadtree, as depicted in Fig. 3, is used as the basis for multiscale modeling.

The prediction step of the Kalman filter incorporates the process dynamics, carrying the estimation statistics from one time to the next, however the computational complexity and storage requirements become problematic for large state dimensions; the size of the SST problem makes it impossible to directly compute predicted estimates and associated error statistics.

Given our choice of multiscale models in the update step, we can consider directly predicting the models themselves[1], however this approach, although promising, is for the time being similarly computationally intensive. Alternatively, in this work, we assume that the ocean dynamics are very slow, so that a very simple

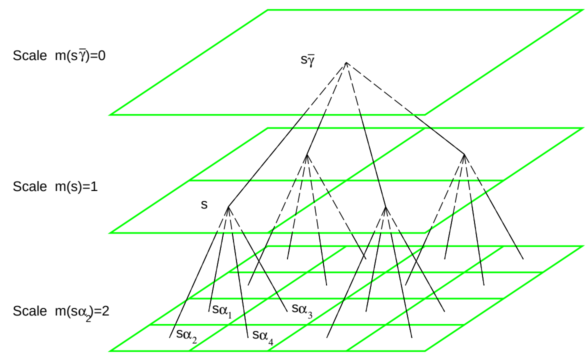


Figure 3: An example multiscale quad-tree for modeling 2D processes.

dynamic model (each pixel independently evolves randomly) is adopted. This model implies the following simple estimate prediction

$$\hat{x}(t|t-1) = \hat{x}(t-1|t-1) \tag{3}$$

It remains to propagate the estimation error statistics. The prior model is embedded in the selected multiscale model, so explicit changes to the prior at every time step are inconvenient; instead, we exploit the *duality* between priors measurements, modifying the prior implicitly by introducing new "measurements." Specifically, the measurements at time $t$ consist of one or two independent components: the satellite SST measurement (if any) and the predicted estimates from the previous time step,

$$\begin{aligned} y'(t) &= \begin{bmatrix} y(t) \\ \hat{x}(t|t-1) \end{bmatrix}, \\ R'(t) &= \begin{bmatrix} R(t) & 0 \\ 0 & f(\tilde{P}(t-1|t-1)) \end{bmatrix} \end{aligned} \tag{4}$$

where $R(t)$ is the covariance of the SST measurement error, and $f(\cdot)$ represents a simplified prediction step. Since only the diagonal elements of $\tilde{P}$ are readily available, we define $f$ as

$$f(\tilde{P}(t-1|t-1)) = \alpha \, \mathrm{diag}(\tilde{P}(t-1|t-1)) + Q \tag{5}$$

where $Q$ is the process noise covariance. Because the predicted covariance is diagonal, (4) treats the errors in the predicted estimates as independent, which is clearly incorrect, since the errors are correlated. Instead, a positive constant $\alpha$ is introduced to inflate the predicted variances, such that the impact of the inflated variances of the independent measurements is comparable to the desired uninflated, correlated ones.

## RESULTS

We applied our recursive filtering algorithm to one month of ATSR night-time data, with a time step of

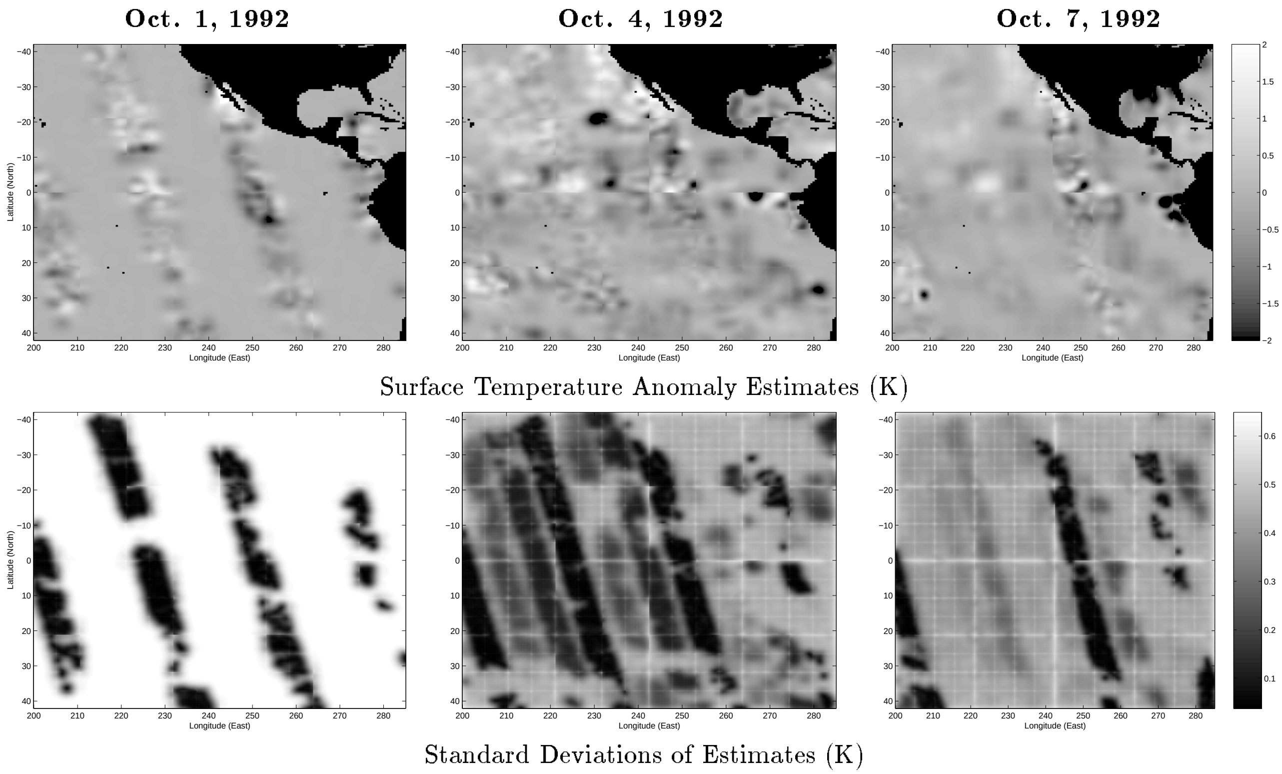


Figure 4: Dynamic Estimation of SST based on night-time observations, three days apart. The upper row is the anomaly estimates of SST and the lower row is their corresponding standard deviations

one day. Based on empirical assessments, we adopted a Gaussian-shaped spatial correlation structure for the SST. Three of the dynamic estimation results and the associated error standard deviations, selected from days 1, 4, and 7, are shown in Fig. 4. All results are shown with respect to a seasonal mean.

In terms of the estimates, we observe at day 1 strictly *local* effects, based on the limited number of measurements; over time the majority of the field is affected. We also observe the fine detail present in "fresh" estimates (based on recent measurements), off the California coast on Oct. 4, which appear less detailed and diffused three days later, after three dynamic prediction steps.

The lower three panels plot the standard deviations of the estimation errors. The interpretation of the leftmost frame is trivial, since no dynamics have yet taken place. Examining the later frames, we see clear evidence of the incorporation of measurements over time, reflected in the different variances of the different measurement swaths: recently measured areas correspond to a low error variance, with the error variance increasing for increasingly older measurements.

The artifacts apparent in the results, particularly in the error statistics, are due to approximations introduced by the multiscale models. We have investigated several ways of overcoming such artifacts, and will shortly apply these to the SST problem.

## References

[1] T. Ho, P. Fieguth and A. Willsky, "Recursive Multiscale Estimation Of Space-Time Random Fields," *ICIP'98*, Chicago, 1998.

[2] C.T. Mutlow and A.M. Zavody, "Sea surface Temperature Measurements by the along-track scanning radiometer on the ERS1 Satellite: Early results," *Journal of Geophysical Research*, Vol. 99, No. C11, pp.22575-22588, 1994.

[3] P. Fieguth, W. Karl and A. Willsky, "Multiresolution Optimal Interpolation and Statistical Analysis of TOPEX/POSEIDON Satellite Altimetry," *IEEE Trans. on Geoscience and Remote Sensing*, Vol. 33 No. 2, pp. 280-292, 1995.

[4] D. Menemenlis, P. Fieguth and A. Willsky, "Adaption of Fast Optimal Interpolation Algorithm to the Mapping of Oceanographic Data," *Journal of Geophysical Research*, vol. 102, No. C5, pp. 10573-10583, 1997.