

# MOTION ESTIMATION OF SPARSE, REMOTELY-SENSED FIELDS

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## ABSTRACT

Sea surface temperature (SST) can be estimated from remotely-sensed images. Because of the sparsity of the available observation it is ideal to do estimation using dynamic methods (such as Kalman filtering). To model dynamics of SST accurately we need to know motion of sea current .

The traditional video motion estimation problem is straightforward, in some ways, because there are so few constraints. That is, the motion vectors are pretty much arbitrary, and successive image frames are densely pixellated, have the same number of pixels, with similar noise statistics. However there are many motion estimation problems, particularly in the area of remote sensing, which do not share these properties.

In this paper we investigate the problem of determining the motion field of the sea surface, based on infrared measurements of surface temperature. This problem is challenging in that only a subset of the whole domain is measured at each point in time; specifically, only a few stripes are imaged. In addition, because of clouds, the measured subset varies from time to time; in fact, some days absolutely nothing is imaged. The quality (level of noise) can also vary from pixel to pixel.

Our research will be based on the following assumptions and observations: the motion field should be smooth and ideally divergence-free, i.e. the motion field is close to time-stationary. Based on these assumptions we choose to use optical flow method for this motion problem. We handle difficulty of data sparsity by pre-estimation to get a dense field. Pre-estimation can be refined by integrating this motion estimation result. Preliminary experiment result will be shown in the end.

## 1. INTRODUCTION

To track the changes of sea surface temperature (SST) we need to dynamically estimate temperature field from remotely-sensed images. For this purpose accurate model of sea dynamics is required.

There are two main factors to concern in modelling SST: temperature diffusion and current motion. Diffusion model

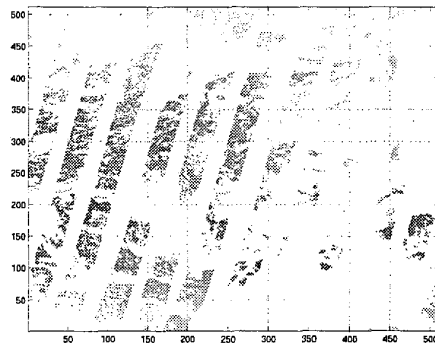


Figure 1 – Sparse Field of SST

has been studied extensively and has already been employed in a Kalman filtering scheme ([2]). However because of some difficulties, especially data sparsity, motion estimation for SST is still an open problem. Here data sparsity means that usually only a subset of the whole domain is measured at each point time in time; specifically, only a few stripes are imaged. In addition, because of clouds, the measured subset varies from time to time; in fact, some days absolutely nothing is imaged. The quality ( level of noise) can also vary from time to time.

In this paper we discuss a motion estimation method for SST field. Based on the assumption that this motion field is smooth and close to be time-stationary we propose using optical flow algorithm with smoothness constraint for motion estimation. To handle sparsity problem we first use diffusion-based Kalman filter to get a dense estimate of the field. Then we apply optical flow method to this dense field.

## 2. DYNAMIC ESTIMATION OF SST FIELD

The dynamic evolution of SST is assumed to obey the linear discrete dynamic model given by:

$$x(t+1) = Ax(t) + w(t) \quad (1)$$

where  $w(t) \sim N(0, Q)$  is an uncorrelated Gaussian noise process, with zero mean, diagonal covariance  $Q$ . The ATSR measurements are linearly related to SST:

$$y(t) = C(t)x(t) + u(t) \quad (2)$$

where  $u(t) \sim N(0, R)$  is an uncorrelated Gaussian noise process, with zero mean, diagonal covariance  $R$ . For physical systems given by models (1), (2), the Kalman filter can be used to obtain filtered estimates for the state  $x(t)$  at time  $t$  based on the data available up to time  $t$ .

The need to generate and invert enormous matrices makes the brute-force implementation of the Kalman filter for ATSR images computationally infeasible for the indefinite future. A multiscale method is employed in ([2]) to overcome this difficulty.

### 3. OPTICAL FLOW WITH SMOOTHNESS CONSTRAINT

Based on the observation that motion in SST field is smooth and slow we choose optical flow method with smoothness constraint to determine this motion field. If we use  $f(x_1, x_2; t)$  to represent image sequences we can write optical flow equation (OFE) as follows,

$$\frac{\partial f(x_1, x_2; t)}{\partial x_1} v_1(x_1, x_2; t) + \frac{\partial f(x_1, x_2; t)}{\partial x_2} v_2(x_1, x_2; t) + \frac{\partial f(x_1, x_2; t)}{\partial t} = 0 \quad (3)$$

where  $v_1 = dx_1 / dt$  and  $v_2 = dx_2 / dt$ .  $(v_1, v_2)$  is the expected motion vector.

There are two unknowns  $(v_1, v_2)$  in OFE. We can not find unique solution of  $(v_1, v_2)$  from it. This is usually called aperture problem.

Several different methods have been proposed to handle aperture problem. Here we use Horn and Schunck's method([1]).

Horn and Schunck seek a motion field that satisfies OFE with the minimum pixel-to-pixel variation among the flow vectors. Specifically they minimize a weighted sum of the error in OFE and a measure of the pixel-to-pixel variation of the velocity field

$$\min_{v_1, v_2} \int_A (\mathcal{E}_y^2(v_1, v_2) + \alpha^2 \mathcal{E}_i^2(v_1, v_2)) dx_1 dx_2 \quad (4)$$

to estimate the velocity vector at each point  $(x_1, x_2)$ , where  $A$  denotes the continuous image support. The parameter  $\alpha^2$  controls the strength of the smoothness constraint. In the above expression

$$\mathcal{E}_y = \frac{\partial f}{\partial x_1} v_1 + \frac{\partial f}{\partial x_2} v_2 + \frac{\partial f}{\partial t} \quad (5)$$

and

$$\mathcal{E}_i^2 = \left(\frac{\partial v_1}{\partial x_1}\right)^2 + \left(\frac{\partial v_1}{\partial x_2}\right)^2 + \left(\frac{\partial v_2}{\partial x_1}\right)^2 + \left(\frac{\partial v_2}{\partial x_2}\right)^2 \quad (6)$$

Horn and Schunck used Gauss-Seidel iteration to solve the above minimization problem. Readers are referred to [1] for the detail of their algorithm. Here we just list the necessary formula for iteration.

Motion vector  $(v_1, v_2)$  is determined by

$$\hat{v}_1^{(n+1)} = \bar{v}_1^{(n)} - \frac{\frac{\partial f}{\partial x_1} \bar{v}_1^{(n)} + \frac{\partial f}{\partial x_2} \bar{v}_2^{(n)} + \frac{\partial f}{\partial t}}{\alpha^2 + \left(\frac{\partial f}{\partial x_1}\right)^2 + \left(\frac{\partial f}{\partial x_2}\right)^2} \quad (7)$$

$$\hat{v}_2^{(n+1)} = \bar{v}_2^{(n)} - \frac{\frac{\partial f}{\partial x_1} \bar{v}_1^{(n)} + \frac{\partial f}{\partial x_2} \bar{v}_2^{(n)} + \frac{\partial f}{\partial t}}{\alpha^2 + \left(\frac{\partial f}{\partial x_1}\right)^2 + \left(\frac{\partial f}{\partial x_2}\right)^2} \quad (8)$$

where  $\bar{v}_1$  and  $\bar{v}_2$  are local averages of estimated velocity.

$$\bar{v}_1 = \frac{1}{6} \{v_1(i-1, j, k) + v_1(i, j+1, k) + v_1(i, j-1, k) + v_1(i+1, j, k) + v_1(i-1, j-1, k) + v_1(i-1, j+1, k) + v_1(i+1, j+1, k) + v_1(i+1, j-1, k)\} \quad (9)$$

$$\bar{v}_2 = \frac{1}{6} \{v_2(i-1, j, k) + v_2(i, j+1, k) + v_2(i, j-1, k) + v_2(i+1, j, k) + v_2(i-1, j-1, k) + v_2(i-1, j+1, k) + v_2(i+1, j+1, k) + v_2(i+1, j-1, k)\} \quad (10)$$

Partial derivatives are estimated by

$$\frac{\partial f}{\partial x_1} \approx \frac{1}{4} \{f(i+1, j, k) - f(i, j, k) + f(i+1, j+1, k) - f(i, j+1, k) + f(i+1, j, k+1) - f(i, j, k+1) + f(i+1, j+1, k+1) - f(i, j+1, k+1)\} \quad (11)$$

$$\frac{\partial f}{\partial x_2} \approx \frac{1}{4} \{f(i, j+1, k) - f(i, j, k) + f(i+1, j+1, k) - f(i+1, j, k) + f(i, j+1, k+1) - f(i, j, k+1) + f(i+1, j+1, k+1) - f(i+1, j, k+1)\} \quad (12)$$

$$\frac{\partial f}{\partial t} \approx \frac{1}{4} \{f(i, j, k+1) - f(i, j, k) + f(i+1, j, k+1) - f(i+1, j, k) + f(i, j+1, k+1) - f(i, j+1, k) + f(i+1, j+1, k+1) - f(i+1, j+1, k)\} \quad (13)$$

Usually the initial values of  $(v_1, v_2)$  are chosen to be zero. And  $\alpha$  is selected heuristically.

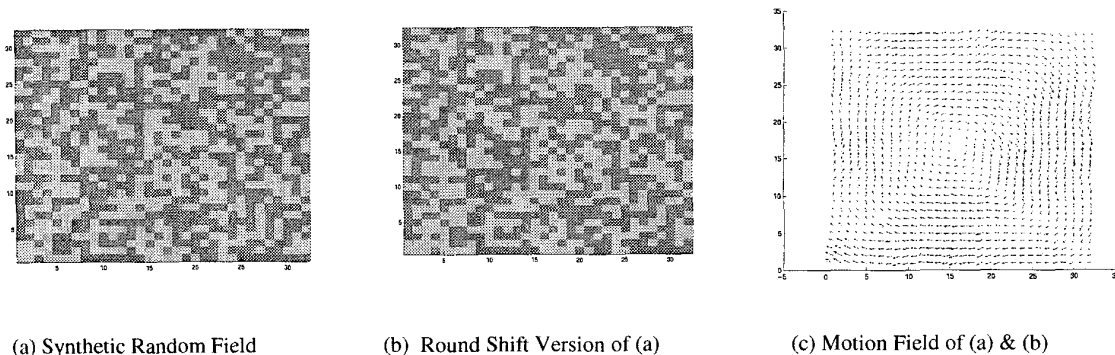


Figure 2 Motion Estimation for Ocean Synthetic Data

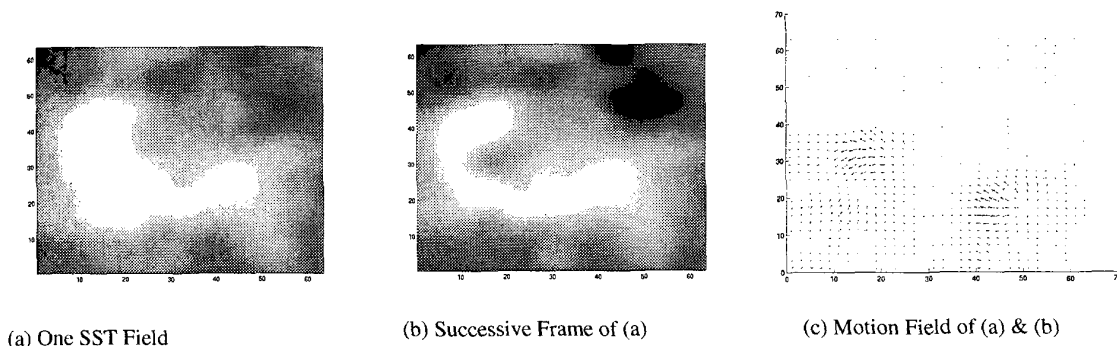


Figure 3 Motion Estimation for Dense SST Estimates

#### 4. EXPERIMENT RESULTS

In the following we show 2 experiment results, one for synthetic image data and the other for ATSR image.

First we apply Horn and Schunck algorithm to a synthetic stationary random field with round shift (Fig.2(a) and Fig. 2(b)). This synthetic data is designed to simulate SST field with a correlation length of 30. Fig.2(a) is the original data. Fig.2(b) is the round shift version of Fig.2(a). The shift is one pixel. Fig.2(c) shows the result of motion estimation. For this experiment  $\alpha$  is set to be 40. The iteration number is 50. Please note that to better estimation of partial derivatives we use a smoothing filter before estimation.

Now we try to apply Horn and Schunck algorithm([1]) to ATSR images. Because usually ATSR images are sparse we need filter them to get dense fields. Fig.3(a) and Fig.3(b) are 2 estimates of dense fields. We use the algorithm in [2] for these estimates. Fig.3(c) is the corresponding motion estimation results by Horn and

Schunck algorithm. For this experiment  $\alpha$  is set to be 40. The iteration number is 50. The result is consistent to our visual observations.

#### 5. SUMMARY

The method for determining optical flow with smoothness constraint by Horn and Schunck has been used to find motion in SST field. To handle data sparsity problem we employed Kalman filter([2]) to get dense field. We hope to use the result of motion estimation to refine dynamic model of SST field and thus improve performance of Kalman filter.

Noting that the estimate of dense field is noisy and this makes estimated motion field at some points unreliable we plan to use information from multiple frames to filter noise in motion estimate. This filter can be achieved in a dynamic form.

## 6. REFERENCES

- [1] B.K.P.Horn and B.G.Schunck, "Determining Optical Flow," *Artificial Intelligence*, Vol. 17, pp185-203, 1981.
- [2] P.Fieguth, F.M.Khellah, M.R.Allen and M.J.Murray, "Large Scale Dynamic Estimation of Ocean Surface Temperature," *Proceeding of IEEE 1999 International Geoscience and Remote Sensing Symposium, Germany*, Vol.3, pp. 1826-1828, 1999.
- [3] A.M.Tekalp, *Digital Video Processing*, Prentice Hall, 1995
- [4] P.Fieguth, M.R.Allen and M.J.Murray, "Hierarchical Methods for Global-Scale Estimation Problems," *CCECE*, pp161-164, 1998