MULTIRESOLUTION MODEL DEVELOPMENT FOR OVERLAPPING TREES VIA CANONICAL CORRELATION ANALYSIS[1]

Paul W. Fieguth William W. Irving Alan S. Willsky

M.I.T. Laboratory for Information and Decision Systems
77 Massachusetts Ave.
Cambridge, MA 02139

ABSTRACT

Recently a class of multiscale stochastic models has been introduced in which Gaussian random processes are described by scale-recursive dynamics that are indexed by the nodes of a tree. One of the primary reasons the framework is useful is that it leads to an extremely fast, statistically optimal algorithm for least-squares estimation in the context of 2-D images. In this paper, we refine this approach to estimation by eliminating the visually distracting blockiness that has been observed in previous work. We eliminate the blockiness by discarding the standard assumption that distinct nodes at a given level of our tree correspond to disjoint portions of the image domain; as a consequence of this simple idea, a given image pixel may now correspond to several tree nodes. We develop tools for systematically building overlapping-tree multiscale representations of prespecified statistics, and we develop a corresponding estimation algorithm for this processes. In this way, we achieve nearly optimal estimation results, we generate corresponding error covariance information, and we eliminate blockiness without sacrificing the resolution of fine-scale detail.

I. INTRODUCTION

Recently, a class of multiscale stochastic models has been introduced in which Gaussian stochastic processes are indexed by the nodes of a tree [1], [2]. These models provide a systematic and powerful way to describe random processes and fields that evolve in *scale.*

The primary reason that the framework is useful is that it leads to extremely efficient, statistically optimal algorithms for signal and image processing; these algorithms exploit the special statistical structure of our models in much the same way that the Kalman filter exploits the structure of Gauss-Markov time-series models. In fact, a particularly successful example of a multiscale-based estimation algorithm is a direct generalization of both the Kalman filter and the related Rauch-Tung-Striebel smoother[2]. Applications for which this approach has met with considerable success include calculation of optical flow[5] and the smoothing of ocean altimetric data[3].

In spite of the success of the multiscale approach with regard to computational efficiency, mean-square estimation error, and ability to supply error covariance information, the approach arguably suffers two limitations:

1. Automatic multiscale model development tools are required.
2. The smoothed estimates produced by the multiscale smoother tend to exhibit a visually distracting blockiness.

Each of these two limitations were researched independently with some success. A recently undertaken merging of these two streams of research has yielded remarkable results, simultaneously achieving three objectives:

1. It yields low-dimensional multiscale models that are quite faithful to prespecified random field covariance structure to be realized, and thus admits an extremely efficient, optimal (or nearly optimal) estimation algorithm;
2. The resulting estimation algorithm retains one of the most important advantages of the multiscale estimation framework, namely the efficient computation of estimation error covariances;
3. Both the multiscale models and the corresponding estimation algorithm eliminate the blockiness associated with previously developed multiscale models and estimates.

II. MULTISCALE FRAMEWORK

The multiscale framework of interest in this paper consists of scale-recursive models defined on index sets that are organized as multilevel trees such as the one shown

[1] P.W.F. was supported by a NSERC-67 fellowship of the Natural Sciences and Engineering Research Council of Canada. W.W.I. was supported by the Lincoln Laboratory Staff Associate Program. Further support provided by the Office of Naval Research under Grant N00014-91-J-1004, by the Draper Laboratory under Grant DL-H-467133, and by ARPA under Grant F49620-93-1-0604.

0-8186-7310-9/95 $4.00 © 1995 IEEE

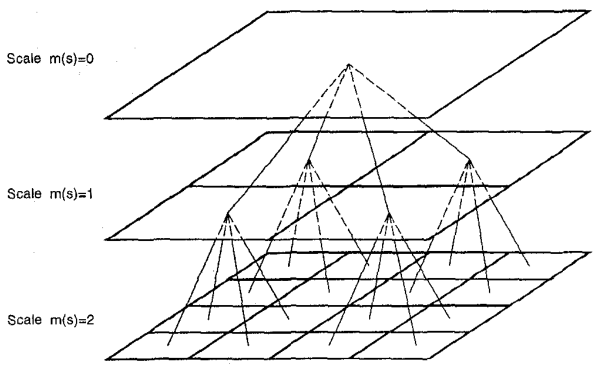

Fig. 1. Example multiscale tree structure.

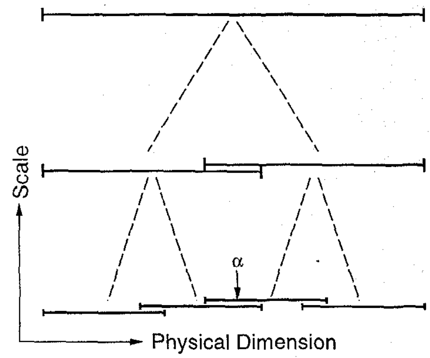

Fig. 2. Simple illustration of an overlapping tree. The point 'α' will have a redundant representation on the finest scale of this tree.

in Figure 1. Here each level of the tree corresponds to a different scale of resolution in the representation of the random field, with the resolutions proceeding from coarse to fine as the tree is traversed from top to bottom.

Let s denote any node on the tree and $s\bar{\gamma}$ its parent; the state elements at these nodes are related by a coarse-to-fine recursion:

$$x(s) = A(s)x(s\bar{\gamma}) + B(s)w(s) \quad (1)$$

where $w(s)$ is a white Gaussian noise process with identity covariance. The multiscale model also permits measurements that are arbitrarily distributed in both space and scale:

$$y(s) = C(s)x(s) + v(s) \quad (2)$$

where $C(s)$ is a matrix specifying the nature of the process observations and $v(s)$ is white with covariance $R(s)$.

III. MODEL DEVELOPMENT

By appropriately defining the information which is conveyed by $x(s)$, and by an appropriate choice of the $A(s)$ and $B(s)$ matrices, we can realize any desired correlation structure for the finest-scale Gaussian process. A conceptually simple model-building strategy leads to the specification of these quantities. To describe the strategy, we first note that any given node on a qth order tree can be viewed as a boundary between $q+1$ subsets of nodes, where q of these subsets correspond to paths leading towards offspring and one corresponds to a path leading towards the parent. With this boundary notion in mind, the strategy can be stated as follows: always retain just enough information in $x(s)$ to ensure the conditional independence of the finest-scale portions of the corresponding $q+1$ subsets of the process. We select a multiscale model such that $x(s)$ contains suitable linear functionals of the finest-scale process to achieve this $q+1$-way conditional independence.

In general, exact realization of desired statistics will require a very high order multiscale model. However, by suitably generalizing the work in [4], [5], we have developed a systematic and broadly applicable procedure for building reduced-order, approximate models of any desired fidelity. This procedure is based on a novel application of the techniques of canonical correlation analysis, and allows us to effectively confront the model complexity/algorithm speed tradeoff.

IV. ARTIFACT ATTENUATION

The presence of visually distracting artifacts or blockiness in the multiscale estimates does not imply an error in the estimation process, but rather implies the selection of a multiscale model which inadequately preserves correlations across the boundaries of the multiscale tree.

In certain applications such blockiness might be eliminated by the simple application of a low pass filter. Unfortunately this approach can ambiguate the interpretation of error covariance information; moreover it limits the resolution of fine-scale details in the post-processed estimate.

To understand our new approach for eliminating blockiness, consider again the multiscale tree of Figure 1. At any given node s in the tree process, the state $x(s)$ represents an aggregate description of the subset of the random field attributed to the node; normally the nodes on a given scale each aggregate disjoint portions of the random field. In our proposed approach to multiscale estimation we relax this constraint and construct processes in which points in physical space may correspond to numerous fine scale tree nodes. We refer to such a tree process as an *overlapped* tree process, because adjacent multiscale nodes are now permitted to represent overlapping regions of the random field.

For example, consider a dyadic tree representation for a 1-D signal as shown in Figure 2. The bracket ⊢⊣ at

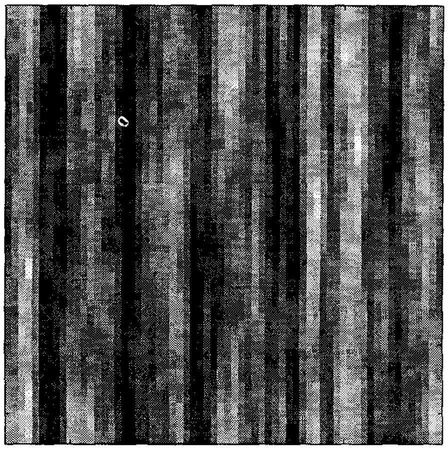

Fig. 3. Original "wood" texture, 64×64 samples, simulated using Markov Random Field techniques. Measurements of this field are corrupted to 0dB SNR.

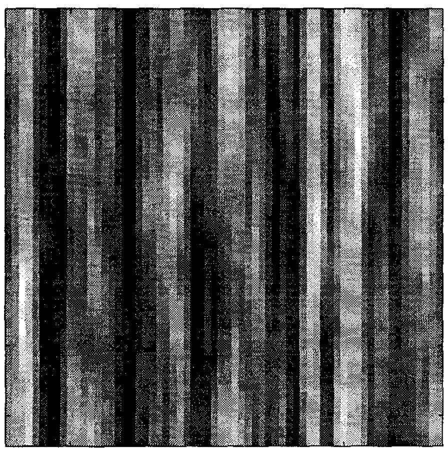

Fig. 4. Estimated texture using optimal FFT techniques, based on noisy measurements of Figure 3.

each node denotes the region of physical space to which is represented by the state vector at that node (e.g., the physical point α is present in two of the intervals on the bottom level of Figure 2). Clearly this leads to a redundant representation at the finest scale of the tree, i.e., several finest scale nodes may correspond to the same point in physical space.

It is the redundancy of the overlapping framework which allows us to obtain a smoother set of estimates. If we let H represent the projection from the redundant finest scale of the overlapped tree into physical space

$$\hat{X} = H\hat{X}_{\text{OVERLAPPED}} \tag{3}$$

then with the appropriate choice of H, multiscale artifacts present in $\hat{X}_{\text{OVERLAPPED}}$ can be eliminated. It is important to realize, however, that H has the effect of an ensemble average – no spatial averaging of any sort is occurring.

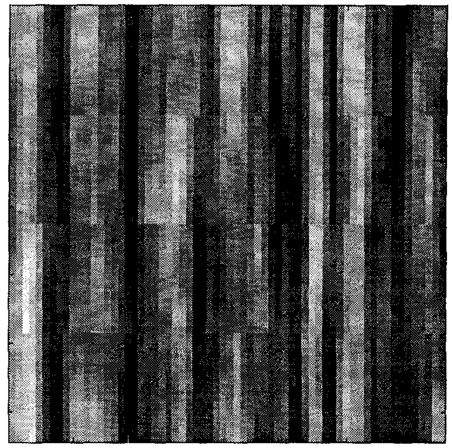

Fig. 5. Estimated texture using a multiscale tree model, but without using an overlapping tree. Note the artifacts across the boundaries of the image quadrants.

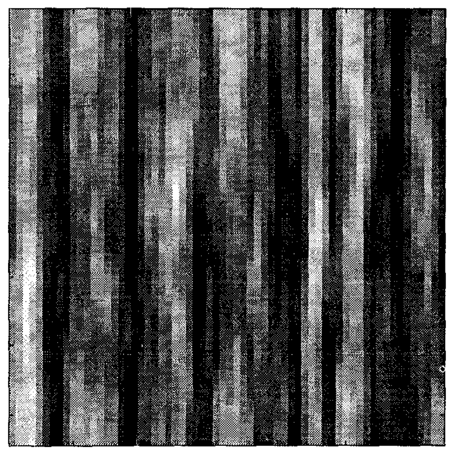

Fig. 6. Estimated texture using a multiscale tree model applied to an overlapped tree. The computational burden of this estimator is the same as that in Figure 5.

V. EXPERIMENTAL RESULTS

We will apply our multiscale estimator to the texture shown in Figure 3; it is based on a Markov random field model possessing considerable correlation in the vertical direction. The image was corrupted to 0dB SNR by white Gaussian noise, and estimated in three different ways:

1. Using an optimal FFT technique (Figure 4)
2. Using a non-overlapped multiscale model of order 40 (Figure 5)

3. Using an overlapped multiscale model of order 16 (Figure 6)

Although the computational burden of the latter two techniques is the same, the estimates of Figure 6 are clearly superior.

The advantages of our multiscale technique over the FFT method stem from the fact that our approach can tolerate irregularly sampled data, spatially varying measurement noise, and a spatially varying prior model – any of which render the FFT approach inapplicable.

A second estimation example is illustrated in Figures 7–9. This example computes the estimates for a random field having a nonstationary prior model; i.e., FFT techniques are no longer applicable. Figure 7 shows a sample path of the nonstationary model. The 64x64 pixels of the process were divided into groups g_1 and g_2: g_1 contains the pixels in the upper left and lower right of the image, and g_2 contains the pixels in the diagonal band running through the center of the image. The prior model for g_1 is the "wood" model of before; the prior model for g_2 is just a rotation of the "wood" texture by 90 degrees. The cross correlation between groups g_1 and g_2 is zero.

Figure 8 shows a noisy version of the original sample path, corrupted by white Gaussian noise to 0dB; Figure 9 shows the corresponding multiscale reconstruction based on an overlapping multiscale model of order $k = 32$. Two observations should be made:

- As mentioned in the previous example, the smoothing operation H_x of the overlapping framework has not at all blurred the edge between the two prior models — the edge stands out distinctly.
- Essentially no artifacts are visible along the correlated bands in either orientation.

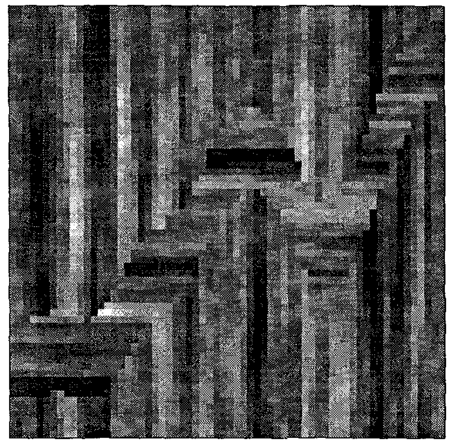

Fig. 7. Inhomogeneous Markov random field sample path.

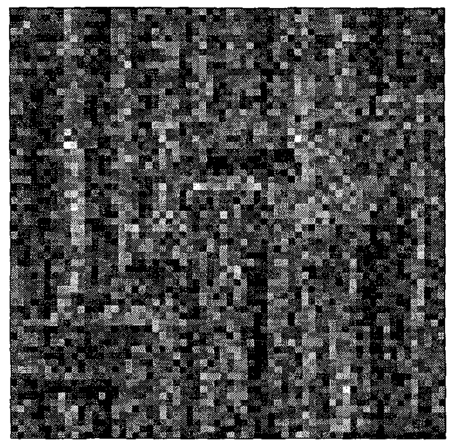

Fig. 8. Figure 7 plus 0dB white, Gaussian, noise.

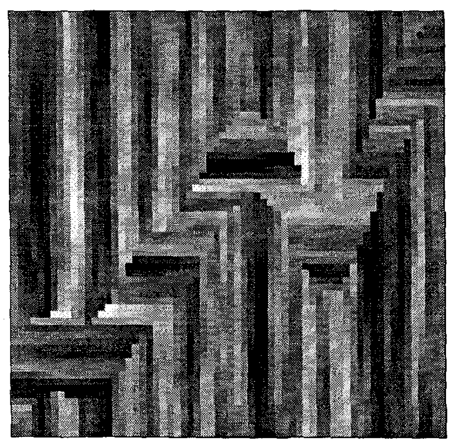

Fig. 9. Overlapped-tree estimates based on Figure 8.

References

[1] M. Basseville et. al., "Modeling and estimations of multiresolution stochastic processes.", *IEEE Transactions on Information Theory* (38) #2, pp.766–784, 1992

[2] K. Chou, A. Willsky, A. Benveniste, "Multiscale Recursive Estimation, Data Fusion, and Regularization", *IEEE Trans. on Automatic Control* (39) #3, pp.464–478, 1994

[3] P. Fieguth, W. Karl, A. Willsky, C. Wunsch, "Multiresolution Optimal Interpolation and Statistical Analysis of TOPEX/POSEIDON Satellite Altimetry", *IEEE Trans. Geoscience and Remote Sensing*, March 1995

[4] W. Irving, W. Karl, A. Willsky, "A Theory for Multiscale Stochastic Realization", *33rd Conference on Decision and Control*, Orlando, FL, December, 1994.

[5] M. Luettgen, W. Karl, A. Willsky, "Efficient Multiscale Regularization with Applications to the Computation of Optical Flow", *IEEE Trans. on Image Processing* (3) #1, pp.41–64, 1994.