MULTISCALE METHODS FOR THE SEGMENTATION OF IMAGES

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ABSTRACT

This work presents a method for segmenting images based on gradients in the intensity function. Past approaches have centered on formulating the problem in the context of variational calculus as the minimization of a functional involving the image intensity and edge functions. Computational methods for finding the minima of such variational problems are prone to two shortfalls: they are often computationally intensive and almost always incapable of computing error statistics associated with the segmentation. Using a particular variational formulation as a starting point, this paper presents a derivation of an associated statistical formulation using multiscale models. The result is an algorithm which is fast and capable of computing error statistics.

1. INTRODUCTION

Many applications require the segmentation of images. In disciplines such as medical imaging and remote sensing, this is often done painstakingly by hand. To reduce the tedium of this component of image analysis, one would like to create automatic image segmentation algorithms. This paper presents a novel segmentation algorithm which belongs to the class of algorithms that segment an image by decomposing it into smooth regions bounded by curves on which the image intensity changes abruptly.

Blake and Zisserman [1], Mumford and Shah [2], Shah [3], and Pien and Gauch [4] have all written about aspects of this approach to segmentation and have proposed various complex functionals whose minima correspond to segmented images. Although the sound reasoning leading to the proposition of the functionals makes their use appealing, the algorithms that find the minima are often computationally intensive and do not provide information about the uncertainty in one's estimate of the location of an edge. Knowledge of such uncertainty is useful in many applications as it can help guide the interpretation of the segmented image.

In this paper, we take an alternative approach to solving the minimization: we determine the least-squares estimation equations corresponding to the minimization of the functionals proposed in [3, 4], and then use a fast multiscale technique [5] to solve the estimation problems. The multiscale approach has the advantage of being not only computationally efficient but also capable of generating error statistics.

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2. GRADIENT SEGMENTATION

The specific functional under consideration is [3, 4]

$$E = \int\int_{\Omega} \left((f-g)^2 + \lambda|\nabla f|^2(1-s)^2 + \frac{\nu}{2}(\rho|\nabla s|^2 + \frac{s^2}{\rho})\right) \quad (1)$$

where Ω is the domain of the image; g represents the observed image; ρ, ν, and λ are scalar constants; and (f, s) is a pair of functions over which one minimizes E. For the pair of functions $(\hat{f}, \hat{s})$ which minimize E, $\hat{f}$ is a piecewise smooth approximation of the observed image g, and $\hat{s}$ is an edge map whose range is the interval $[0,1]$. A region boundary is declared at those locations where $\hat{s} \approx 1$.

The algorithms discussed in [3, 4] perform a coordinate descent in order to minimize (1). They alternate between fixing s and minimizing

$$E_s = \int\int_{\Omega} ((f-g)^2 + \lambda|\nabla f|^2(1-s)^2) \quad (2)$$

over possible f, and fixing f and minimizing

$$E_f = \int\int_{\Omega} \left(\lambda|\nabla f|^2(1-s)^2 + \frac{\nu}{2}(\rho|\nabla s|^2 + \frac{s^2}{\rho})\right) \quad (3)$$

over possible s. Thus, the problem of minimizing the complex functional (1) is reduced to repeatedly minimizing the two convex functionals (2) and (3). With each of these minimization problems, one can associate a statistical estimation problem. The motivation for seeking such an equivalent estimation formulation is that one can then use a fast multiscale estimation algorithm to perform the minimization and to generate error statistics. First, one-dimensional problems will be considered in order to simplify the analysis. Two-dimensional extensions will be presented in Section 4.

3. SEGMENTATION IN ONE DIMENSION

3.1. The Estimation Problem

In one-dimension, (2) becomes

$$E_s = \int_\Omega \left((f-g)^2 + \lambda |\frac{df}{dx}|^2 (1-s)^2 \right) dx. \qquad (4)$$

The problem of minimizing this functional with respect to f is equivalent to estimating f given the following measurements and prior model:

$$\begin{aligned} g(x) &= f(x) + v^f(x) && (5) \\ \frac{df(x)}{dx} &= \frac{1}{(1-s(x))\sqrt{\lambda}} w^f(x) && (6) \end{aligned}$$

where $v^f(x)$ and $w^f(x)$ are independent Gaussian white-noise processes with unit intensity.

The one-dimensional version of (3) is

$$E_f = \int_\Omega \left(\lambda |\frac{df}{dx}|^2 (1-s)^2 + \frac{\nu}{2} (\rho |\frac{ds}{dx}|^2 + \frac{s^2}{\rho}) \right) dx. \quad (7)$$

With the substitutions

$$\begin{aligned} a(x) &= \lambda |\frac{df}{dx}|^2 & b &= \frac{\nu}{2\rho} \\ \gamma(x) &= \frac{a(x)}{a(x)+b} & c &= \frac{\nu\rho}{2}, \end{aligned} \qquad (8)$$

minimizing (7) is equivalent to minimizing

$$E_f = \int_\Omega \left((a+b)(s-\gamma)^2 + c|\frac{ds}{dx}|^2 \right) dx. \qquad (9)$$

If one ignores the constraint that $s(x) \in [0,1]$, then the minimization of (9) can be rewritten as the following estimation theoretic problem:

$$\begin{aligned} \gamma(x) &= s(x) + \frac{1}{\sqrt{a(x)+b}} v^s(x) && (10) \\ \frac{ds(x)}{dx} &= \frac{1}{\sqrt{c}} w^s(x) && (11) \end{aligned}$$

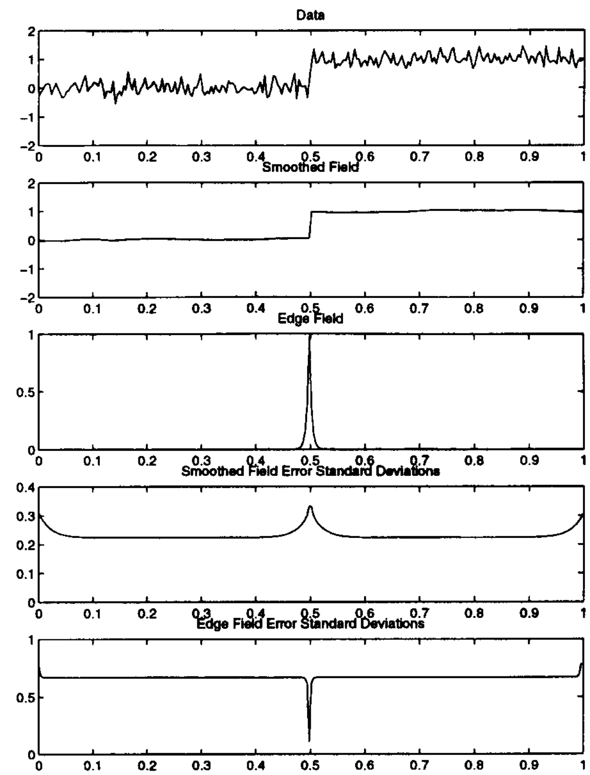

Figure 1: An example of segmenting a step edge with 0.2 standard deviation Gaussian noise added to the measurements.

where $w^s(x)$ and $v^s(x)$ are independent Gaussian white-noise processes with unit intensity. The resulting estimate of s is truncated after each iteration so that it lies in $[0,1]$.

3.2. Results

Figure 1 shows the results of a one-dimensional segmentation experiment. The top three panels show respectively the function g, a step with noise added; the estimated piecewise-smooth function f; and the estimated edge process s, where $s \approx 0$ (or 1) represents the absence (or presence) of an edge in the data. The results are quite promising, accurately detecting step edges and exhibiting robustness to additive noise.

In each case, the function pair (f, s) is estimated by alternately solving the estimation problems (5),(6) and (10),(11); both estimation problems are solved exactly on a multiscale tree using a model similar to that of [5]. The estimation algorithm generates not only estimates but also the error standard deviations, which are displayed in the bottom of figure 1.

The convergence of the iterative scheme is quite fast. The results for figure 1 are taken after seven iterations, beyond which point further iterations do not produce any significant changes. Three iterations bring one close to the final stopping point. Interestingly, the different initial estimates we tried yielded the same final estimate.

4. EXTENSIONS TO TWO DIMENSIONS

4.1. The Estimation Problem

Motivated by these results, we are in the process of extending the one-dimensional segmentation algorithm to two dimensions. The primary challenge is the development of two-dimensional estimators: the estimation problems being solved in one-dimension do not have exact two-dimensional equivalents that can be solved with the multiscale methods. In order to make use of these techniques, one must derive multiscale two-dimensional analogues of (5),(6) and (10),(11). Fieguth develops a line of approach on how to do this in his Ph.D. thesis [6], and the ensuing discussion uses this work as a starting point.

In order to make use of the multiscale estimation algorithm, one needs to formulate the problem in terms of a recursive stochastic model on a tree. The trees used for image processing are quad trees: each node has four descendants. An abstract index ν is used to specify a particular node on the tree, and the notation $\nu\bar{\gamma}$ is used to refer to the parent of node ν (see figure 2). The process that lives on the tree has a state variable x_ν at every node and is defined by the root-to-leaf recursion

$$x_\nu = A_\nu x_{\nu\bar{\gamma}} + B_\nu w_\nu \tag{12}$$

where the w_ν and the state x_0 at the root node are a collection of totally independent zero-mean Gaussian random variables, the w's with identity covariance and x_0 with some prior covariance. The A and B matrices are deterministic quantities that define the statistics of the process on the tree. Observations y_ν of the state variables have the form

$$y_\nu = C_\nu x_\nu + v_\nu \tag{13}$$

where the v_ν are a totally independent collection of Gaussian random variables, and the matrices C_ν are deterministic quantities that specify what is being observed.

Within this framework, one can write down recursive equations, analogous to the one-dimensional estimation problems for segmentation (5), (6) and (10), (11). First notice that equations (6) and (11) may be discretized by replacing the derivative with a difference such as $(f(x+1)-f(x))$ and $(s(x+1)-s(x))$ respectively. This leads to a recursive description for the

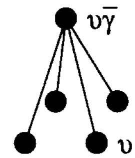

Figure 2: On a quad tree, if ν is the index of some node, $\nu\bar{\gamma}$ denotes the parent of that node.

stochastic process in one-dimension. A natural way to extend this form to the tree is to replace the first differences in one variable with first differences between scales. Then, the problem of smoothing the image given the discontinuities becomes:

$$g_\nu = f_\nu + v_\nu^f \tag{14}$$

$$(f_\nu - f_{\nu\bar{\gamma}}) = \frac{1}{(1-s_\nu)\sqrt{\lambda}d_\nu} w_\nu^f, \tag{15}$$

where d_ν are a set of constants that decrease geometrically as the scale becomes finer[6]. Likewise, one could reformulate the problem of determining the edge field given a smoothed image as:

$$\gamma_\nu = s_\nu + \frac{1}{\sqrt{a_\nu + b}} v_\nu^s \tag{16}$$

$$(s_\nu - s_{\nu\bar{\gamma}}) = \frac{1}{\sqrt{c d_\nu}} w_\nu^s \tag{17}$$

where

$$a_\nu = \lambda(f_\nu - f_{\nu\bar{\gamma}})^2 \tag{18}$$

$$\gamma_\nu = \frac{a_\nu}{a_\nu + b}. \tag{19}$$

Notice that the principal novelty of this two-dimensional formulation is the use of the first difference between scales in lieu of a local difference operator acting in a plane.

The preceding discussion specifies a pair of multiscale statistical problems and associated estimation operators that one can iteratively apply to segment an image. The operators, though, are not applied directly to the image. Instead, one projects the image into an overlapped domain, where one can apply the estimation operators, and then projects the results back to the original domain[7]. The final algorithm is diagrammed in figure 3.

4.2. Results

Figure 4 displays the results for employing the preceding procedure to segment a synthetic, noisy image of a circle. The ratio of discontinuity size to noise standard deviation is 10. A threshold of 0.9 was used to create the thresholded edge field. The images displayed

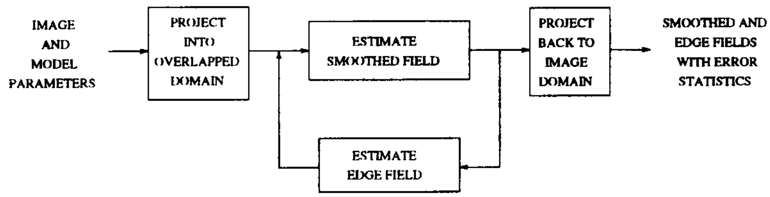

Figure 3: Flow Diagram for the multiscale segmentation algorithm.

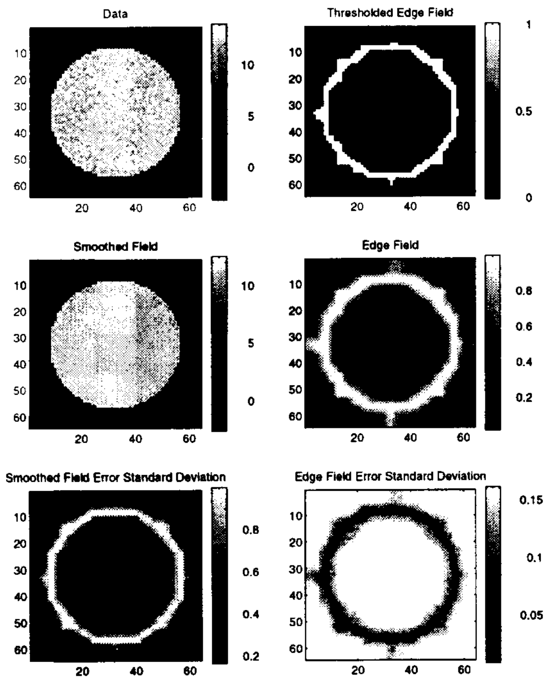

Figure 4: An example of segmenting a synthetic circle image with measurement noise. The ratio of discontinuity size to noise standard deviation is 10

are taken after five iterations, beyond which point further iterations do not produce significant changes. The results indicate that the algorithm can pick out the boundary of the circle, despite the presence of noise, and also generate the error variances that are an indication of the quality of the estimates of the smoothed and edge fields

5. CONCLUSIONS AND FURTHER WORK

The multiscale estimation problem derived from the variational formulation of Shah shows promise for use as a method for segmenting images. The segmentations produced by the algorithm are visually meaningful, and the algorithm is both computationally efficient and capable of generating error statistics. Although this paper has not explored how the statistics may be used, such statistics are useful for many applications. One direction of further research is the investigation into the meaning and uses of the error statistics obtained from the multiscale algorithm. The results of such research are liable to show that the multiscale approach to segmentation is suitable for many problems.

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