Tensor Vector Field Based Active Contour

UNIVERSITY OF Waterloo
uwaterloo.ca

Vision and Image Processing Group
Systems Design Engineering
vip.uwaterloo.ca

Abhishek Kumar, Alexander Wong, Akshaya Mishra, David Clausi, and Paul Fieguth

a42kumar@uwaterloo.ca a28wong@uwaterloo.ca akmishra@engmail.uwaterloo.ca dclausi@engmail.uwaterloo.ca pfieguth@uwaterloo.ca

Introduction:

- Active Contours are a set of methods in which a contour is initialized near an object of interest, and the contour converges to the boundary of the object.
- The convergence is achieved by iteratively minimizing the energy of the contour.
- The energy of the active contour is guided by the following two rules.
  - Internal energy: The contour should maintain some shape constraint. Usually constraints are put against stretchability and high curvature.
  - External energy: There should be a field created by the object of interest which can attract a contour toward its boundary.
- Two main challenges of active contours are:
  - Sensitivity to image noise.
  - Limited capture range for the contour.
- A variety of external fields have been introduced to address these limitations, two of the more significant and standard approaches are
  - Gradient Vector Field (GVF): Increased active contours capture range, but computationally expensive and sensitive to noise.
  - Vector Field Convolution (VFC): Handled limitations of GVF to a great extent, but does not take full advantage of the structural property of the image boundary.

Objective:

We propose a tensor vector field (TVF) by adapting the VFC kernel using the structural information of an image. This results in more stable field in the presence of noise and thus provides improved active contours.

Vector Field Convolution:

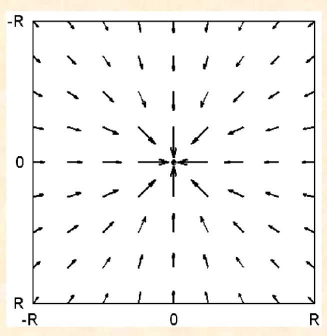
The contour deforms with the iterative minimization of its energy equation, expressed as:

$$E_{AC} = \int_0^1 \left[E_{int}(c(s)) + E_{ext}(c(s)) \right] ds$$

E_{int} is the internal energy and E_{ext} is the external energy. In VFC, the external field is generated by convolving the image edge-map with an isotropic vector field kernel **k** ($R \times R$). The direction of vectors of **k** at each position (i, j) is given as,

$$\mathbf{n}(i,j) = [-i/r, -j/r]$$

$$r = \sqrt{i^2 + j^2}$$



Vector Field Kernel

Where, i, j are position with respect to the kernel's center. The magnitude (m) of kernel's each vector element is given by,

$$m(i,j) = (r+\epsilon)^{-\zeta} \text{ or } m(i,j) = \exp -\frac{r^2}{\sigma}$$

Where σ and ζ are positive constants to control decay of m. So **k** can be expressed as,

$$\mathbf{k} = \mathbf{n}(i,j) m(i,j)$$

Tensor Vector Field:

As kernel (**k**) of VFC does not use structural information of the image, it gets affected by image noise. In TVF direction and magnitude of each of the kernel element is modified using image tensor Γ, so that the well structured object boundaries can create a relatively unperturbed external field. Γ can be expressed for each pixel (x, y) as,

$$\Gamma_{x,y} = \begin{pmatrix} \sigma_{x,x} & \sigma_{x,y} \\ \sigma_{y,x} & \sigma_{y,y} \end{pmatrix}$$

$$\sigma_{x,y} = \sum_{i=-\kappa/2}^{\kappa/2} \sum_{j=-\kappa/2}^{\kappa/2} g(i,j) u_x(i,j) u_y(i,j)$$

σ_{xy} is a weighted variance or co-variance matrix for each pixel (x, y) in the image, g is a Gaussian mask (κ× κ) used to compute σ_{xy}.

During convolution each element of the kernel **k** is modified using the major eigenvalue (λ_+) and major eigenvector (v_+) of Γ as given below.

$$\mathbf{k}_{mod}(i,j) = |\mathbf{n}(i,j) \cdot \mathbf{v}_+(i,j)| m(i,j) \lambda_+(i,j) \mathbf{n}(i,j)$$

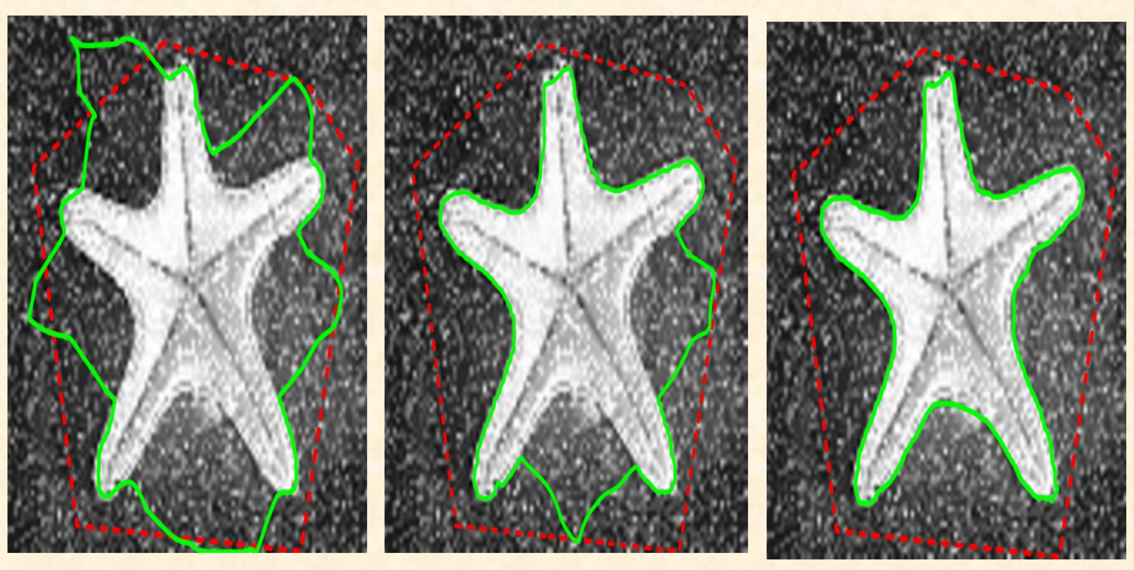
Experimental Results:

- Results of the proposed TVF based active contours have been compared with two standard external fields: GVF and VFC.
- Experiments are shown for the starfish image.

External field Streamline:

- The image is corrupted with impulsive noise.
- A high density and low irregularity of the field lines (shown in black) due to noise implies a better pulling of the active contour towards the boundary.

Active Contour:

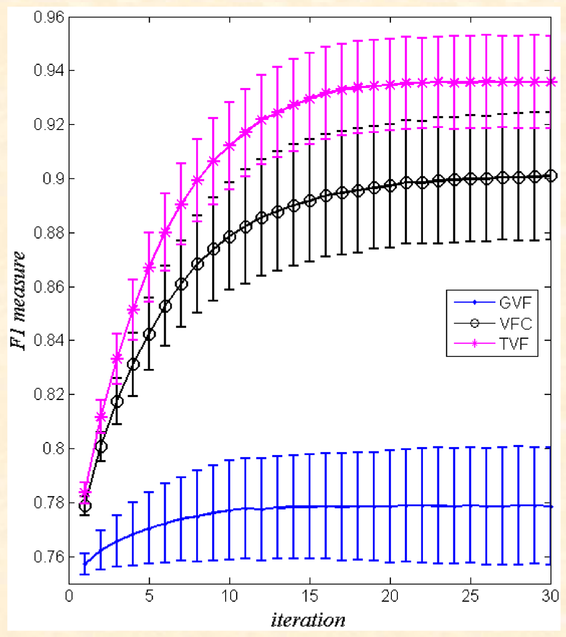


- The image is corrupted by Gaussian noise (PSNR = 25.5db).
- The initial active contour (red) is converged to final contour (green) after 30 iterations.

Experimental Results (Cont.):

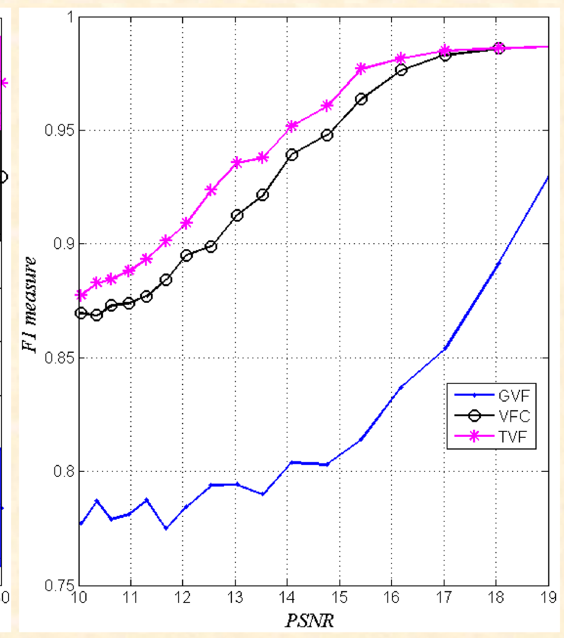
- Results are shown for the starfish image, corrupted with Gaussian noise.
- Twenty runs are used to collect consistent statistics.
- Results are compared using the F1-measure[1].

F1-measure vs. Iteration



- PSNR 12.7dB
- TVF provides more accurate active contours then VFC without sacrificing convergence rate.

F1-measure vs. PSNR



- For a wide range of noise (PSNR) the TVF leads to the best accuracy.

F1-measure for other sample images.

Image	PSNR (dB)	F1 measure(mean ± standard deviation)		
		GVF	VFC	TVF
	12.7	0.79 ±0.022	0.90 ±0.024	0.93 ±0.017
	12.6	0.90±0.017	0.95 ±0.013	0.97 ±0.006
	12.6	0.92±0.013	0.97 ±0.009	0.98 ±0.001
	12.9	0.85±0.022	0.93 ±0.028	0.97 ±0.021
	12.6	0.84±0.035	0.92 ±0.021	0.96 ±0.026

- F1-measure is computed at the end of 30 iterations for image samples, contaminated with Gaussian noise.
- TVF based active contours consistently outperforms the other two.

1. F1-measure ranges from 0 to 1 and higher F1-measure implies better result.

Note: A small correction has been made with respect to the publication, where in few results PSNRs were quoted double of the actual value.

Conclusions:

- TVF based external field stream lines are clearly least affected by noise. The active contour based on the TVF energy function surrounds the given object more accurately then other two methods tested.
- The active contours guided by TVF takes about the same number of iterations as VFC to converge but gives consistently better active contour.
- With TVF, we consistently obtain a better accuracy for a wide range of image noise.

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