Simultaneous Gamma Correction and Registration in the Frequency Domain

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Abstract ***This paper presents a novel method for performing the registration and gamma correction of images in a simultaneous manner. The image registration and relative gamma estimation problems are formulated such that both problems are solved at the same time by solving a single minimum squared distance problem. The proposed method utilizes the Fast Fourier Transform (FFT) to solve the combined registration and relative gamma estimation problem in an efficient manner. This method is invariant to gamma differences between images, making it suitable for registering images acquired under different exposure levels and illumination conditions. Experimental results demonstrate that high registration accuracy and high relative gamma estimation accuracy can be achieved using the proposed algorithm.***

Keywords: registration, gamma correction, frequency domain, simultaneous.

1 Introduction

Image registration is the process of aligning images of the same scene acquired from different devices, perspectives, or under different conditions. Techniques for image registration have played an important role in a large range of applications, such as panoramic photo generation [1], medical image fusion [2], and the alignment of remote sensing images [3]. In the traditional approach to image registration, pairs of matching control points are manually selected from the images being registered. These matching control points are then used to estimate a transformation that brings the images into alignment. This approach is very time consuming given that control point pairs must be chosen manually by a user. Therefore, methods for registering images in an automated fashion are desired.

Research into automated image registration has produced a wide range of techniques. In general, these methods can be grouped into the following categories:

1. **Intensity-Based Techniques [3–5]**: Intensity-based techniques determine the alignment between images by comparing images directly based on their pixel intensity values. Such techniques are typically used for aligning images acquired using the same device and under very similar conditions. An example of this is panoramic generation, where a series of images are acquired from the same scene at different perspectives and stitched together into a single panoramic image.

2. **Frequency-Based Techniques [6, 7]**: Frequency-based techniques use frequency domain characteristics such as phase to determine the alignment between two images.

3. **Gradient Feature-Based Techniques [8–11]**: Gradient feature-based techniques determine the alignment between images by comparing images based on features extracted from local gradient characteristics. Common gradient-based features include edges [8], gradient orientation [9,10], and corners [11]. Gradient feature-based techniques are useful in scenarios where the images are acquired under very different conditions but distinctive structural details exist in the images begin registered.

4. **High-Level Feature-Based Techniques [12]**: High-level feature-based techniques determine the alignment between images by matching high-level features such as regions, shapes, and semantic objects (e.g., buildings and roads). These techniques are effective for aligning images that have distinctive objects or known shapes.

To the best of the authors' knowledge, there are currently no techniques available that perform both image registration and gamma correction simultaneously in an efficient and robust manner. Such an approach is particularly important for situations where images are acquired under different exposure levels and illumination conditions. The registration accuracy of existing techniques typically decreases under such situations, since shared image content in overlapping regions is represented using different intensity values due to variations in exposure and illumination conditions. While gamma correction has been proposed as a pre-processing step for image registration to address this issue, it is difficult to determine the relative gamma value accurately between two images with shared content without knowing the actual overlapping region. Since registration must be performed to determine the overlapping region, a circular dependency is created between image registration and gamma correction. The goal of the proposed algorithm is to address this issue in an efficient and robust manner.

The main contribution of this paper is a novel approach that performs simultaneous image registration and gamma correction in an efficient manner using the Fast Fourier Transform (FFT). The proposed method is highly robust against gamma differences between images, and therefore suitable for performing image registration and gamma correction on images acquired under different exposure levels and illumination conditions. In this paper, the underlying theory behind the proposed algorithm is discussed in Section 2. The proposed algorithm is described in Section 3. The methods and data used to test the effectiveness of the algorithm are outlined in Section 4. Experimental results are presented and discussed in Section 5. Finally, conclusions are drawn in Section 6.

2 Theory

Before outlining the proposed algorithm, it is important to present the theory behind the key concepts of the algorithm. First, the theory behind gamma correction is described. Then, the formulation of the registration and relative gamma estimation problems as a single minimum squared distances problem is presented. Finally, the theory behind solving the simultaneous registration and relative gamma estimation problem efficiently in the frequency domain is presented.

2.1 Gamma Correction

Gamma correction is a non-linear method for controlling the overall luminance of a still image. Gamma correction is typically used to adjust still images for accurate reproduction on physical displays (e.g., computer monitors and TV displays). In this case, gamma correction is used to compensate for the non-linear relationship between pixel values and the intensity being displayed. Images that are not gamma corrected may appear washed out or overly dark. Gamma correction may be expressed by the following power-law relationship:

$$I_{\text{corrected}} = I_{\text{uncorrected}}^{\gamma} \tag{1}$$

where $I_{\text{uncorrected}}$ and $I_{\text{corrected}}$ are the uncorrected and corrected images respectively, and γ is the gamma value. If $\gamma < 1$, the overall luminance range of the uncorrected image is compressed in a non-linear fashion. This is often referred to as a gamma compression. If $\gamma > 1$, the overall luminance range of the uncorrected image is expanded in a non-linear fashion. This is often referred to as a gamma expansion. An example of both gamma compression and gamma expansion is illustrated in Figure 1.

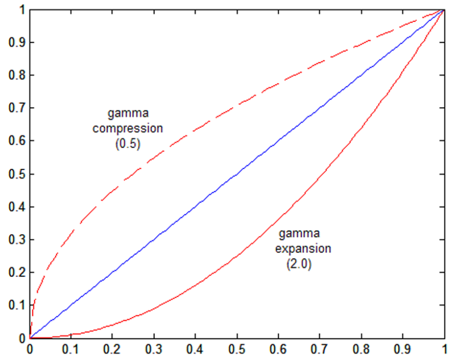

Figure 1: Example of gamma compression and gamma expansion

Similarly, gamma correction can be used to improve image consistency in a series of related images. It is often the case where images captured of the same scene have noticeable illumination variations. Therefore, gamma correction is desired to maintain illumination consistency amongst related images. This is particularly important in situations where the images are used to generate panoramas, where consistent illumination conditions are required to obtain an aesthetically pleasing panoramic image. An example of a panoramic image constructed from uncorrected images is illustrated in Figure 2. A significant distinction between the individual images can be seen, resulting in a panoramic image that is not pleasing to the eye.

Figure 2: Example of panoramic image generated using uncorrected images.

The common approach to obtaining illumination consistency amongst a series of images is to designate one of the images in the series as a reference image (usually the image with best visual quality) and determine the relative gamma values that relate the reference image and the rest of images. It is essential that there exists overlapping content between the reference image and the target images, as relative gamma must be determined in regions of shared content. In the case where the series of images consists of two images, let g represent the reference image and f represent the target image to be corrected. Based on the power-law relationship described in (1), the non-linear relationship between the reference image and the target image can be expressed by:

$$R_g = R_f^{\gamma} \tag{2}$$

where R_g and R_f are the overlapping regions in the reference and target images respectively, and γ is the relative gamma value between the two images. By determining the relative gamma between the two images, the target image can be corrected using the following formula:

$$f_{\text{corrected}} = f^{\gamma} \tag{3}$$

2.2 Simultaneous Registration and Relative Gamma Estimation

Given two images f and g, the images may be registered to each other by evaluating the similarities between the two images and then determining the alignment that produces the maximum similarity between overlapping regions. One approach for evaluating similarity is to compute the sum of squared distances between two images as follows:

$$C = \sum_{\underline{x}} (f(\underline{x}) - g(\underline{x}))^2 R(\underline{x}) \tag{4}$$

where $\underline{x}$ is the two-dimensional coordinate of a pixel within an image and $R(\underline{x})$ is a function that defines a specific region of interest in g if the approximate overlapping region is known. A low value of C indicates high image similarity. In the case where the alignment between two images can be expressed by a translation, the optimal alignment between two images can be determined by finding the translation vector that minimizes the sum of squared distances. Therefore, the registration problem can be formulated as a minimum squared distance problem:

$$\underline{t} = \arg\min_{\underline{t}} \left[\sum_{\underline{x}} (f(\underline{x} - \underline{t}) - g(\underline{x}))^2 \, R(\underline{x}) \right] \tag{5}$$

where $\underline{t}$ is optimal translation vector that aligns f with g.

In the ideal case, where the overlapping regions have identical intensity values, the minimum sum of squared distances at the optimal alignment is equal to zero. However, this is seldom the case in real-world situations due to different exposure levels and illumination conditions. Such differences in illumination conditions result in different intensity values in overlapping regions, which lead to registration error. The most common approach to solving this problem is to perform gamma correction on the target images before registration is performed. However, to perform gamma correction between images, it is necessary to know what the overlapping regions are such that a relative gamma value can be determined. This results in a circular dependency between registration and gamma correction. A novel approach to addressing this issue is to perform both registration and relative gamma estimation simultaneously.

To perform registration and relative gamma estimation in a simultaneous manner, it is necessary to formulate a single problem that combines both tasks together. Applying logarithms to both sides of Equation 2, the relationship can be written as follows:

$$\log(R_g) = \gamma \log(R_f) \tag{6}$$

To incorporate this non-linear relationship between overlapping regions, the sum of squared distances between two images f and g can be rewritten as follows:

$$C = \sum_{\underline{x}} \left(\gamma \log(f(\underline{x})) - \log(g(\underline{x}))\right)^2 R(\underline{x}) \tag{7}$$

Using the above similarity metric, the final minimum squared distance problem incorporating both registration and relative gamma estimation is expressed as follows:

$$\underline{t}, \gamma = \arg\min_{\underline{t}, \gamma} \left[\sum_{\underline{x}} \left(\gamma \log(f(\underline{x} - \underline{t})) - \log(g(\underline{x}))\right)^2 R(\underline{x}) \right] \tag{8}$$

2.3 Problem Solving in the Frequency Domain

It can be seen that the final minimum squared distance problem presented in Equation 8 has three degrees of freedom (two for translation and one for the relative gamma value). This problem is computationally expensive to solve in a direct manner, particularly for large images. Therefore, a method to reduce the computational complexity of the minimization problem is desired.

One powerful approach to reducing the computational complexity of the primary problem presented in Equation 8 is to formulate a dual problem that can be solved in an efficient manner. A solution to the dual problem is also a solution to the primary problem. In the proposed algorithm, the dual problem is established by reformulating the primary problem in the frequency domain. This type of approach has been used effectively to significantly reduce the computational complexity of correlation-based problems [4, 10, 13].

The distance metric presented in Equation 7 can be rewritten as follows:

$$\begin{aligned} C = \quad & \gamma^2 \sum_{\underline{x}} \left(f_l(\underline{x} - \underline{t})\right)^2 R(\underline{x}) \\ & -2\gamma \sum_{\underline{x}} \left(f_l(\underline{x} - \underline{t})\right)\left(g_l(\underline{x})\right) R(\underline{x}) + \sum_{\underline{x}} g_l(\underline{x})^2 R(\underline{x}) \end{aligned} \tag{9}$$

where $f_l(x) = \log(f(x))$ and $g_l(x) = \log(g(x))$ Reformulating the terms that are dependent on $\underline{t}$ as convolutions, the distance metric becomes the following:

$$C = \gamma^2 \left(\widehat{f}_l^{\,2} * R\right)(\underline{t}) - 2\gamma \left(\widehat{f}_l * (g_l R)\right)(\underline{t}) + \sum_{\underline{x}} g_l(\underline{x})^2 R(\underline{x}) \tag{10}$$

where $\widehat{f}_l(\underline{x}) = f_l(-\underline{x})$ and $*$ denotes a convolution. The last term in Equation 10 does not depend on $\underline{t}$ and can be removed from the equation without affecting the solution of the minimization problem. The convolution terms in Equation 10 are computationally expensive if evaluated in a direct manner. However, the computational complexity can be significantly reduced by exploiting frequency domain characteristics. An important property that can be exploited is the fact that convolutions in the spatial domain become multiplications in the frequency domain, which are significantly faster to compute. Therefore, the final distance function can be expressed as:

$$C = \gamma^2 F^{-1}\left(F(\widehat{f}_l^{\,2})F(R)\right)(\underline{t}) - 2\gamma F^{-1}\left(F(\widehat{f}_l)F(g_l R)\right)(\underline{t}) \tag{11}$$

where F is the Fourier Transform and F^{-1} is the inverse Fourier Transform. What is particularly interesting about the above formulation is that it effectively evaluates the sum of squared distances for all possible translations simultaneously in an efficient manner. The optimal relative gamma values for each translation can be solved analytically for each translation by finding the minimum of the quadratic in Equation 11. The dual problem is then to determine the translation that minimizes the cost function presented in Equation 11 given these optimal relative gamma values. When used in conjunction with the Fast Fourier Transform (FFT), the dual problem can be solved in a significantly faster fashion than the direct approach.

3 Proposed Algorithm

Utilizing the theory presented, the proposed registration and gamma correction algorithm consists of the following steps:

1. Given a reference image g and a target image f, take the logarithm of both images to obtain g_l and f_l.

2. Solve for the optimal translation $\underline{t}$ and optimal relative gamma γ using the simultaneous registration and gamma estimation process presented in Section 2.3.

3. Correct f with the optimal relative gamma γ using the following formula:

$$f_{\text{corrected}} = f^{\gamma} \tag{12}$$

4. Align $f_{\text{corrected}}$ and g using the optimal translation $\underline{t}$.

4 Testing Methods

To investigate the effectiveness of the proposed algorithm, five sets of images were registered and gamma corrected. Each test set consisted of two photographic images with overlapping regions. A brief description of the test sets is provided below.

1. HONGKONG:
   586 × 480 images of the Hong Kong skyline.

2. QUEBEC:
   1061 × 804 images of Quebec City aerial view.

3. MONTREAL:
   966 × 772 images of a section in the Montreal Biodome.

4. STOCKHOLM:
   1285 × 618 images of scenery in Stockholm.

5. ALGARVE:
   617 × 463 images of scenery in Algarve.

To evaluate the performance of the proposed algorithm in both registration and gamma correction, each of the target images in the test sets was altered such that the relative gamma between overlapping regions was at a specific known value. The relative gamma values $\gamma = 5/6$ and $\gamma = 9/5$ were used for testing purposes. The alignment error and the relative gamma estimation error were then calculated based on the alignment and gamma estimation results. The alignment error between two images was calculated as the Euclidean distance from the estimated translation to the actual translation between the two images. The relative gamma estimation error was calculated as the difference between the estimated relative gamma and the actual relative gamma between the two images.

5 Experimental Results

A summary of the registration accuracy and gamma estimation accuracy for the test sets is presented in Table 1. It can be observed that the alignment errors and gamma estimation errors are low for all test sets. This demonstrates the effectiveness of the proposed algorithm for simultaneously performing registration and gamma correction on photographic images. Examples of registered and corrected images are shown in Figure 3, Figure 4, Figure 5, and Figure 6.

6 Conclusions

This paper has introduced a novel method for simultaneous image registration and gamma correction in the frequency domain. Experimental results show that a high level of registration accuracy and relative gamma estimation accuracy can be achieved using the proposed method. It is our belief that this method can be used successfully to align and correct images acquired under different exposure levels and illumination conditions. Future work involves extending the proposed method for situations where the illumination conditions vary locally within overlapping regions, as well as situations where the alignment between images must be represented using a non-rigid transformation model.

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Table 1: Registration and Gamma Estimation Accuracy for Test Images

Test Set	**Mean Alignment Error**		**Gamma Estimation Error**	
	$\gamma = 5/6$	$\gamma = 9/5$	$\gamma = 5/6$	$\gamma = 9/5$
HONGKONG	1.0	1.40	0.007	0.005
QUEBEC	1.0	1.00	0.008	0.003
MONTREAL	2.0	2.24	0.009	0.004
STOCKHOLM	1.0	1.00	0.002	0.002
ALGARVE	1.4	1.40	0.003	0.001

Figure 3: MONTREAL test set images; Top-Left: Reference, Bottom-Left: Target, Right: Result

Figure 4: HONGKONG test set images; Top-Left: Reference, Bottom-Left: Target, Right: Result

Figure 5: STOCKHOLM test set images; Top-Left: Reference, Bottom-Left: Target, Right: Result

Figure 6: QUEBEC test set images; Top-Left: Reference, Bottom-Left: Target, Right: Result

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