Spectrum Sensing in Cognitive Radio Using a Markov-Chain Monte-Carlo Scheme

Xiao Yu Wang, Alexander Wong, and Pin-Han Ho

***Abstract*—In this letter, a novel stochastic strategy to spectrum sensing is investigated for the purpose of improving spectrum sensing efficiency of cognitive radio (CR) systems. The problem of selecting the optimal sequence of channels to finely sensing is formulated as an optimization problem to maximize the probability of obtaining available channels, and is then subsequently solved by using a Markov-Chain Monte-Carlo (MCMC) scheme. By employing a nonparametric approach such as the MCMC scheme, the reliance on specific traffic models is alleviated. Experimental results show that the proposed algorithm has the potential to achieve noticeably improved performance in terms of overhead and percentage of missed spectrum opportunities, thus making it well suited for use in CR networks.**

***Index Terms*—Spectrum sensing, cognitive radio.**

I. Introduction

ONE of the primary objectives of cognitive radio (CR) networks is to enable an efficient utilization of spectrum resources without affecting the performance of primary user networks. Many currently reported research works in CR systems have focused on the topic of spectrum sensing, which is generally considered the first step on the way to medium access. It involves identifying available channels using either cooperative approaches or non-cooperative approaches. A comprehensive survey on spectrum sensing can be found in [1].

In non-cooperative approaches, one of the main challenges is in determining how to perform fine sensing in an efficient and effective manner, in which a set of channels is selected for fine sensing such that can maximize the probability of obtaining available channels. A limitation with existing non-cooperative spectrum sensing approaches is that their performance heavily depends on the accuracy of the assumed parametric traffic model. For example, recent non-cooperative spectrum sensing algorithms in [2]–[4] assume an ON/OFF exponential traffic model and therefore their performance is highly dependent on how well the traffic model matches the real-world behaviour. Hence, the performance of such methods can degrade noticeably when such modeling assumptions do not hold true. However, in CR ad hoc networks with high traffic dynamics and media heterogeneity, it is extremely challenging to achieve precise traffic modeling via parametric approaches, which may cause significant performance degradation in the sensing results. To our best knowledge this has been an open question to the design of non-cooperative sensing schemes, which is taken as a fundamental problem to the implementation of non-cooperative CR sensing schemes.

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In this work, we attempt to solve the problem by investigating the potential of employing a non-parametric approach to spectrum sensing, aiming to alleviate the dependence on the problem from specific parametric traffic modeling. We will show that the proposed approach can achieve noticeably improved performance when compared with existing state-of-the-art sensing schemes in terms of overhead and percentage of missed spectrum opportunities, while maintaining excellent sensing performance under different traffic scenarios. While statistical inference have been previously investigated [5], they are fundamentally different since the proposed method is a stochastic, nonparametric approach for spectrum sensing while that work is a cross-layer framework that uses mean statistics deterministically.

II. System Model

Consider a licensed spectrum containing M non-overlapping channels indexed with $i, i = 1, 2, ..., M$. Note that the M channels are not necessarily equally spaced. The M channels are shared by N_p primary users and N_s secondary users who seek opportunities to access the licensed spectrum resources. At each secondary user, a fast sensing over the M channels is performed regularly (and possibly periodically) by way of energy detection over a wide range of spectrum, where the interval can be set according to IEEE 802.22 functional requirement. For each round of fast sensing, there exist two hypotheses H_1 and H_0, which indicate presence and absence of primary network signals on channel i, respectively. Hence, the probability density function (PDF) of the test statistics of channel i, denoted as u_i, can be expressed as [6]

$$f(u_i) = \begin{cases} \frac{1}{2^{k/2}\Gamma(k/2)} u_i^{(k/2)-1} e^{-u_i/2}, & H_0 \\ \frac{1}{2} e^{-(u_i/2+\mu)} (\frac{u_i}{2\mu})^{k/4-0.5} I_{(k/2)-1}(\sqrt{2\mu u_i}), & H_1 \end{cases}, \quad (1)$$

where k is the degrees of freedom, μ is the instantaneous signal-to-interference-plus-noise ratio (SINR), Γ d enotes the Gamma function, and I denotes a modified Bessel function.

Upon the request of data transmission, a fine sensing process is initiated over the spectrum via a selected sequence of channels based on the fast sensing result of each channel in the previous round. This is to precisely assess channel availability with the intended second receiver.

III. Proposed Algorithm for Fine Sensing

The section introduces our approach based on Markov-Chain Monte-Carlo for dynamic spectrum sensing. We particularly focus on identification of the sequence of available channels via a non-parametric approach, so as to achieve better opportunity in channel access.

A. Problem Formulation

Let $\mathcal{T}$ denote a sequence of time instances and $t \in \mathcal{T}$. Let $\tilde{S}_t$ be a random variable taking on the channel indexes $i, i = 1, 2, ..., M$ of channels to be finely sensed at time t. Let s_t denote the realization of $\tilde{S}_t$ on choosing a particular channel. Furthermore, let X_{s_t} be a binary random variable representing the channel availability of s_t, which takes a value of 1 if the channel is available and 0 otherwise. The problem of selecting channels for fine sensing can be formulated as follows. At time instance ξ, the optimal sequence of channels for fine sensing, denoted as $\{s_{t_1}, s_{t_2}, \cdots, s_{t_j}\}_\xi$, (where $\xi \le t_1 <, ..., < t_j \le \xi + t_{\max}$, and $t_1, ..., t_j$ represent the time instances for starting each channel fine sensing process in an optimal sequence), is to be determined such that the probability of channel availability is maximized within a time limitation t_{max}. Hence, the problem of finding the optimal channel sequence for fine sensing can be formulated as follows:

$$\underset{s_{t_1},\cdots,s_{t_j}}{\arg\max} \left\{ P(X_{s_{t_1}} = 1, \cdots, X_{s_{t_j}} = 1) \right\}. \tag{2}$$

B. Proposed Solution by Markov-Chain Monte Carlo

An important challenge in solving the above formulated problem is the incompleteness of channel status information obtained in the previous fast sensing which causes uncertainty in the channel availability. The Markov-Chain Monte-Carlo (MCMC), as a non-parametric approach, has appeared to be very successful at solving problems characterized by uncertainty. The proposed MCMC approach is based on the Metropolis Hastings algorithm [7]. Let the target probability of a channel s_t being selected for fine sensing, denoted as $P(\tilde{S}_t)$, be defined as

$$P(\tilde{S}_t = s_t) = f(s_t)/K, \tag{3}$$

where $f(s_t)$ is an unnormalized function of s_t that represents the probability of channel availability as jointly discovered by the fast sensing and fine sensing process, and K is a constant that normalizes $f(s_t)$: $\sum_{\forall s_t} P(\tilde{S}_t = s_t) = 1$. However, it is very difficult to obtain a completely accurate channel availability distribution via the fast sensing process in such a highly dynamic and unpredictable radio environment. In this case, a method that can precisely approximate the channel availability distribution is desired.

Due to the dynamic nature of CR networks, the target density $P(\tilde{S}_t)$ should be adaptive to the unnormalized function $f(s_t)$ based on the most updated fast sensing result. Let the instantaneous statistics $P_{inst}(\tilde{S}_t = s_t)$ be defined as the likelihood of channel s_t being available. Using a Rayleigh fading channel model, $f(s_t)$ can be determined as

$$f(s_t) = \frac{1}{\bar{\mu}} \int_0^\infty \int_\lambda^\infty f(u_{s_t}) \exp(-\frac{\mu}{\bar{\mu}}) du_{s_t} d\mu, \tag{4}$$

where $f(u_{s_t})$ is the PDF expressed in Eq. (1) with replaced subscript s_t to denote channel i, $\bar{\mu}$ is the average SINR, λ is determined by the probability of a false alarm. With $f(s_t)$, a sequence of channels for fine sensing can be selected by sampling the target probability $P(\tilde{S}_t)$. Note that directly sampling $P(\tilde{S}_t)$ is difficult, while $P(\tilde{S}_t)$ can be indirectly sampled easily compared with that on a parametric model such as exponential and normal distribution. To sample the distribution of $P(\tilde{S}_t)$, we employ an acceptance-rejection sampling approach. Instead of drawing a sequence of channels $\{s_{t_1}, s_{t_2}, \cdots, s_{t_j}\}_\xi$ at time instance ξ directly on $P(\tilde{S}_t)$, we draw the sequence of channels indirectly from a proposal density $q(s'_{t_k}|s_{t_{k-1}})$. In specific, to determine the k^{th} channel in the sequence, we first draw a channel s'_{t_k} from proposal density $q(s'_{t_k}|s_{t_{k-1}})$ and a sample v from a uniform distribution U(0,1). We then compute the probability of channel selection $\alpha(s'_{t_k}|s_{t_{k-1}})$ based on the previous selected channel $s_{t_{k-1}}$ as,

$$\alpha\left(s'_{t_k}|s_{t_{k-1}}\right) = \min\left\{1, \frac{P(\tilde{S}_t = s'_{t_k}) \cdot q(s_{t_{k-1}}|s'_{t_k})}{P(\tilde{S}_t = s_{t_{k-1}}) \cdot q(s'_{t_k}|s_{t_{k-1}})}\right\}. \tag{5}$$

Using a symmetric proposal density $q(s'_{t_k}|s_{t_{k-1}})$ such as a Gaussian distribution (i.e., $q(s'_{t_k}|s_{t_{k-1}}) = q(s_{t_{k-1}}|s'_{t_k})$) and substituting Eq. (3) into Eq. (5), the constant K from the numerator and denominator cancel each other out, thus,

$$\alpha\left(s'_{t_k}|s_{t_{k-1}}\right) = \min\left\{1, \frac{f(s'_{t_k})}{f(s_{t_{k-1}})}\right\}. \tag{6}$$

The proposal channel s'_{t_k} is then accepted as channel s_{t_k} if $v \le \alpha\left(s'_{t_k}|s_{t_{k-1}}\right)$ and $s'_{t_k} \notin \{s_{t_1}, \cdots, s_{t_{k-1}}\}$. This approach generates a Markov chain in which each selection s_t depends only on the previous selection s_{t-1}.

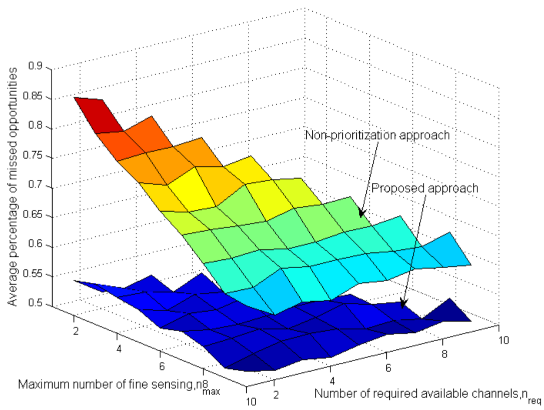
The above process is repeated until the desired sequence of channels for fine sensing $\{s_{t_1}, s_{t_2}, \cdots, s_{t_j}\}_\xi$ at time instance ξ is determined. As such, the fine sensing is performed based on the above sequence of channels and terminated when the requested number n_{req} of available channels are identified, or when the maximum number n_{max} of fine sensing can be performed within time limitation t_{max}. The computational complexity of this proposed method yields $O(M)$ [8].

IV. Performance Evaluation

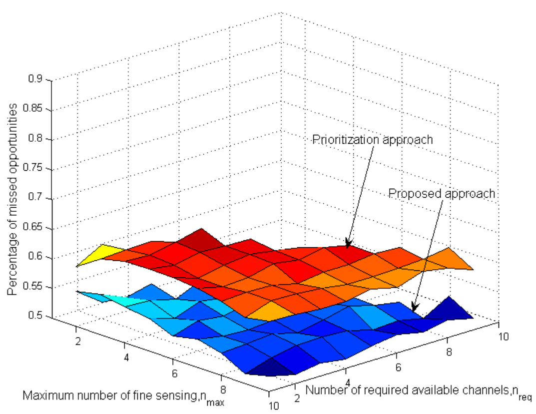
To evaluate the effectiveness of the proposed spectrum sensing algorithm, simulations are conducted to compare the proposed approach with the traditional non-prioritization approach where the channels with the highest probability of channel availability based on fast sensing results are finely sensed, and the state-of-the-art prioritization approach proposed in [4]. In the simulations, we assume 40 channels exist in the licensed spectrum (i.e., $M = 40$). The average SINR is set to 20 dB, and the probability of false alarm is 0.01 [6]. Finally, the probability of detection is computed according to Eq. (4). We observed the proposed method under different randomly generated packet arrival patterns, including: i) exponential packet arrival with an average arrival rate of 10 arrivals/sec, ii) constant packet arrival with an arrival rate of 10 arrivals/sec, and iii) Pareto packet arrival with a Pareto distribution with minimum possible value as 0.01 and index as 1. We first study the percentage of missed spectrum opportunities p_m,

$$p_m = 1 - \frac{n^*_{ava}}{\min\{n_{ava}, n_{\max}\}}, \; n_{ava} \neq 0, n_{\max} \neq 0 \tag{7}$$

where n^*_{ava} is the number of obtained available channels. The performance associated with the different number of request



(a) Proposed approach vs. non-prioritization approach



(b) Proposed approach vs. prioritization approach [4]

Fig. 1. Comparison of average percentage of missed spectrum opportunities of obtaining different number of required available channels n_{req} with different maximum number of fine sensing n_{max}.

available channels n_{req} and the corresponding maximum number of fine sensing n_{max} is shown in Fig. 1. It can be seen that the average percentage of missed spectrum opportunities of the proposed approach is noticeably lower than that achieved by the other tested approaches. The improved performance stems from its ability of better capturing the dynamic nature of spectrum availabilities in CR networks.

Next, we study the average percentage of missed spectrum opportunities under the following traffic models in terms of inter-arrival time: i) exponential, ii) constant, and iii) one form of Pareto, as shown in Fig. 2 for $n_{max} = 8$. The performance of the proposed approach remains consistent under the different traffic scenarios while the performance of the prioritization approach is much more sensitive to the variation of traffic scenarios. To study sensing efficiency, the sensing overhead o is evaluated,

$$o = \begin{cases} \min\{n_{\max}, n_{\Sigma}\} - n^*_{ava} & n^*_{ava} = n_{req} \\ n_{\max} & n^*_{ava} \neq n_{req} \end{cases} \quad (8)$$

A plot of the average overhead in obtaining an available channel (i.e., $n^*_{ava} = 1$) with respect to the maximum number of fine sensing is given in Fig. 3. The average sensing overhead of the proposed approach is noticeably lower than that of the other approaches.

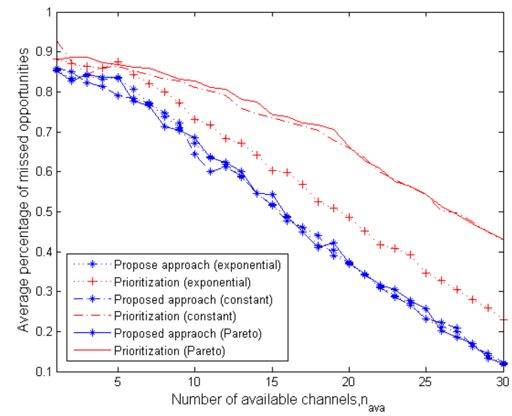


Fig. 2. Comparison of average percentage of missed opportunities of proposed approach over different traffic scenarios

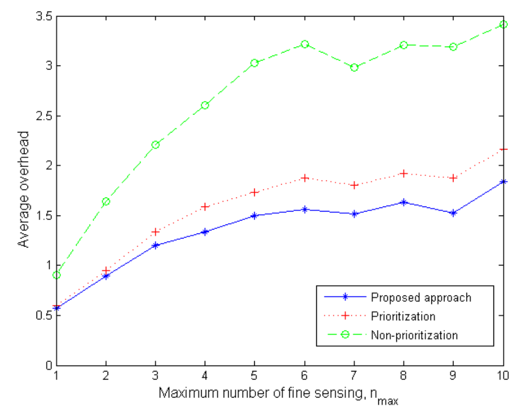


Fig. 3. Average overhead of obtaining an available channel with different maximum number of fine sensing n_{max}.

V. Conclusion

In this letter, we presented a novel spectrum sensing scheme based on a Markov-Chain Monte-Carlo approach. Experimental results demonstrated that the proposed scheme can achieve better sensitivity in the presence of traffic variation noticeably due to the non-parametric design - a desired feature for spectrum sensing schemes in the future dynamic CR networks.

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