Satellite Orbit Determination using Ground-based Navigation Data

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Abstract

There exist various methods of using ground signals collected by a satellite to determine the orbit of the satellite. These signals can be used independently as a single source, or multi-source data can be fused together to increase redundancy and reliability of the orbit solution. A new measurement technique is proposed in this study, in which the location of a satellite could be computed from the knowledge of navigation data points within the satellite's field of view (FoV). There are a growing number of ground-based sensors which broadcast their position, and which can be detected from space, providing a dense dataset for determining the position of the detecting satellite. Knowing the positions of such points, the time that they appear and remain in the FoV, it is possible to determine the satellite's orbit.

In the context of this problem, it is proposed to use ship-based Automatic Identification System (AIS) data as the ground points whose positions are known (with some uncertainty) through the AIS reports from the ships. This AIS navigation data, which are broadcast by the ships and can be acquired by the satellite, can be fed into an orbit determination tool to fit a trajectory for the satellite. A number of simplifying assumptions will be made initially to formulate the approach for deriving the algorithms required to use such measurements. Thereafter, the assumptions will be able to be relaxed successively to provide an analysis technique robust enough to handle real-world data.

I. Introduction

It is often useful and necessary to determine the location of a satellite or constellation thereof, at any point in time. There are many existing methods of using ground signals to determine the orbit of a satellite. These measurements can be used independently as a single source, or multi-source data can be fused together to increase redundancy and reliability of the orbit solution.

A new measurement technique is proposed in this paper: the location of a satellite could be computed from knowledge of the position of points in the field of view of a possible satellite payload. Thus, this basically becomes a reverse orbit determination problem in that knowing the positions of such points and the time that they appear and remain in the FoV, can we determine the satellite's orbit?

In the context of this problem, it is proposed to use ship-based AIS data as the ground points whose positions are known. The navigation data from various sensors onboard the ships, as seen by the satellite, can be fed into an orbit determination tool to fit a trajectory through the error ellipsoids of satellite position. Figure 1 depicts the realization of a scenario over three time epochs (t_0, t_1 and t_2).

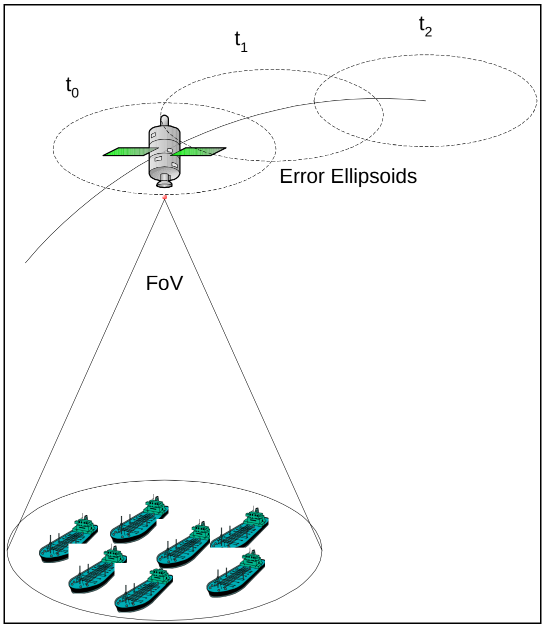

Figure 1: Single Satellite Position Based on Navigation Data of Ships in the Instantaneous FoV of the Satellite

A simplified expression and approximation of the approach to determine the satellite's position from a group of ships in the satellite's field of view (FoV) should be established before assessing the solution's robustness to real world constraints, errors and precision. For example, the satellite FoV and orbit could be assumed circular and it may be assumed there are no errors in the ship positioning information. Figure 2 shows such a scenario where a circular FoV of the satellite encompasses a cluster of ships. The ship cluster here is assumed to extend to the edge of the FoV, which itself is assumed to see out to the limb. In this manner, the locations of the beam extremities could be known via the most separated ship locations. Knowing such locations, the angular separation (α) could be calculated, and using basic trigonometry, satellite altitude (h) obtained from the right-angle triangle shown in the figure, per Equation 1.

$$h = R_E\left(\frac{1}{\cos\alpha} - 1\right) \qquad \text{(Eq. 1)}$$

With three ships at the edge of the field of view, its geometric centre could be determined as a latitude and longitude, to determine the satellite position in three dimensions [2][3].

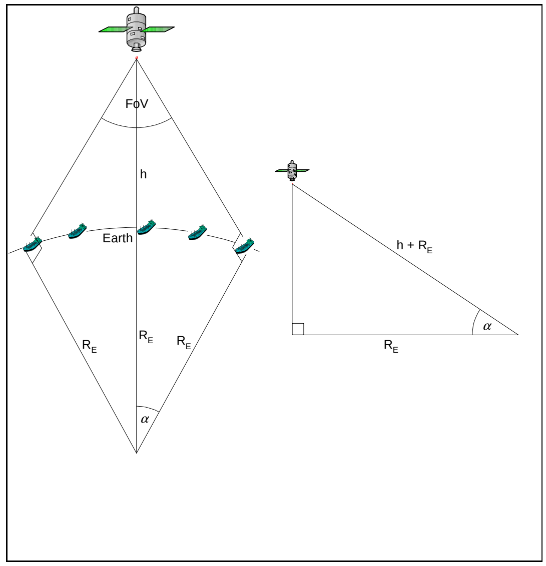

Figure 2: Geometry of the Satellite's FoV Encompassing a Cluster of Ships

II. The Real-world Problem

The above formulation clearly over-simplifies the problem. Modelling the Earth as spherical is clearly far too simplistic. To achieve good orbit knowledge, indeed, it may be necessary to go to extremes of modeling the non-static sea surface height and statistical variation in ship transmission antenna heights. The FoV for a spacecraft sensor able to see out to the limb nominally in all directions may not always see uniformly in all directions, and may well be subject to spacecraft pointing, payload reception pattern variability due to atmospheric or noise variations etc. For a spacecraft payload detecting ship AIS transmissions, the transmissions themselves are discrete at intervals of between 2 and 180 seconds (resulting in imperfect knowledge of start and stop times for ships being in the field of view [3]), and not all transmissions are able to be captured due to the transmissions from multiple ships "colliding" in dense shipping areas. It is certainly unreasonable to assume that at any time instant there will necessarily be multiple ships on the edge of the field of view, simply due to shipping distributions. The encompassing FoV would never be a perfect circle due to the satellite's off-nadir pointing and the ship to satellite link budgets, amongst effects. Variability in satellite altitude would also have to be taken into account.

Ship position measurements themselves have errors and the satellite dynamics with time will be subject to orbital perturbations. These error sources would have to be modeled and integrated in the system. The AIS messages also contain no height information for the ships. The satellite's position in all axes will be subject to a dilution of precision based on the geometry of the ships in the field of view.

The ship position measurements are typically from a variety of onboard navigation aids [1]:

- Position, velocity, time: typically GPS data but could use inertial or Loran-C measurements
- Heading: gyro compass or inertial measurements or GPS
- Rate of turn from rate of turn measurement or inertial measurements
- Speed over ground from sonar measurements or GPS

As can be realized from the above input types, each of the possible input ship position data sources will have different behaviors, characteristics, error patterns, bias, drifts, data rates, accuracies, dimensions, functional models, etc. Thus, each of the data inputs could be custom-fitted and weighted properly in the state model to harvest their respective contributions into the overall solution.

A final complexity to consider is the relative dynamics between the sensors and targets. The capture in the FoV is continually changing over time, and the FoV itself is also changing. Therefore, one must consider the subtleties with formation, maintenance, termination and their respective delays when targets go in and out of the sensors' scope.

On the other hand, there are things that help the estimation process. A large number of ships in the FoV make a statistical orbit determination possible. Since the maximum satellite altitude will not be varying significantly over the course of the orbit, the FoV capture would be fairly consistent. Augmentation could also include the use of fixed points such as ground/base stations and other auxiliary data sources.

Relative dynamics between the sensors and targets reduce system stability, but at the same time furnish valuable information in the time domain. This is why state estimation plays a crucial part in ensuring that the dynamic system is modeled properly and its time evolution is consistent. A robust design of the dynamic model and its time filter should alleviate this burden on the system. This also provides opportunities for improvement in the measurement domain. When dynamic processes are involved with temporal evolution, obviously a time filter approach provides many benefits and offers a means of error propagation. Thus, using a Kalman filter to smooth, predict and/or filter the data would prove very useful in this application. This technique will be especially advantageous in addressing the uncertainty in spacecraft altitude, and when the dynamic model will be corrupted by errors in the ships' positioning domain.

The expansion of the problem could also be done in the target domain, where the ships could be tracked using multiple satellites. As depicted in Figure 3, there could be two or more satellites whose FOVs would contain common ships. Navigation data from these vessels can support determination of the orbits of those satellites. Thus, this would become a multitarget tracking and multisensor information fusion problem. Conceptually, this is therefore a very simple estimation problem to grasp but the intricacies lie in the implementation.

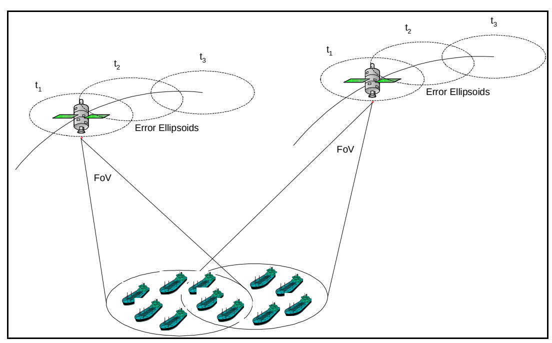

Figure 3: Satellites' Position Based on Navigation Data of Ships in the Instantaneous FoV of the Satellites

III. Overview of the Proposed Approach

To determine the satellite trajectory from ships in the field of view consists of identifying the most likely path such that the ships seen would be seen at the reported times, and not seen before and after these times. The novel approach proposed in this paper is to consider this problem as one of shape recognition in the spatiotemporal domain, namely that of determining the field of view and its trajectory over time, and thereby determining the satellite position. This can be considered as an object recognition problem with a sparse data field providing evidence of the object's shape and position, where the object in question is the satellite's field of view.

A modified Generalized Hough Transform (GHT) is proposed to statistically determine most likely spacecraft trajectory as a function of most likely field of view trajectory that would see the ships whose transmissions are being detected from t to $t + \Delta$. Whilst the data field is only "sparsely" covered, there are a large number of individual ships whose raw measurements are available in both spatial and temporal domains to help identify the most probable satellite trajectory. This can help mitigate for measurement errors and process noise. The advantages of a GHT are their robustness to partial or slightly deformed shapes, and tolerance to noise. The difficulty of using a GHT is in reducing the search space to a manageable size computationally, however for the orbit determination problem, a number of constraints on possible trajectories can be used to prune the space significantly.

To understand the basic concept and illustrate the scenario, let us consider the problem of identifying the satellite position based on a time of observation of a point at a known location, in one dimension only, shown in Figure 4. In this scenario, we have a satellite flying at a fixed height over a surface, with a known and fixed field of view, and it sees a point on the ground at a time t. At that instant in time, the satellite could be at either of the locations shown in Figure 4. If the satellite then sees a second ship further to the right in the plane shown, at time $t + \Delta$, the direction of the satellite is towards the right and the precise trajectory of the satellite can be determined.

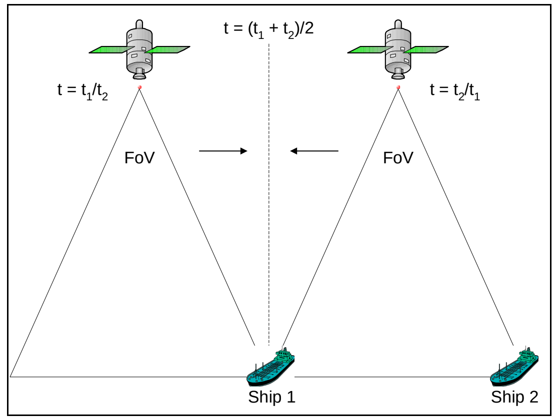

Figure 4: Satellite's Direction Determined by Progression of Visibility in 1-D

For the "two-dimensional" case where the satellite can be at any height (albeit fixed) and any position, in a plane perpendicular to the ground, satellite height can be determined from the fact that only at certain height and one of the two directions, the satellite is able to go from a state of not seeing either ship prior to time t, to seeing only one ship from time t to $t + \Delta$, to seeing two after $t + \Delta$. This scenario is depicted in Figure 5.

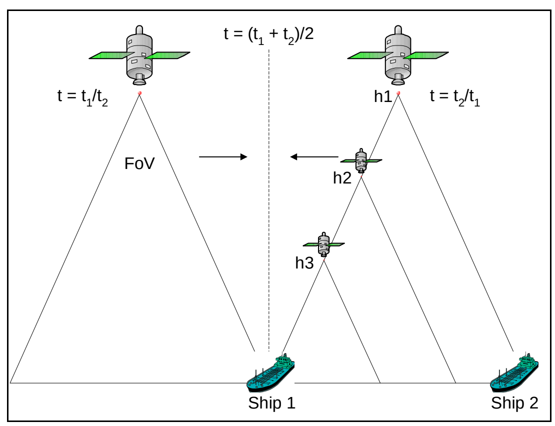

Figure 5: Satellite's Direction and Height Determined by Progression of Visibility in 1-D

Obviously, considering an unconstrained three-dimensional case would make the problem significantly more complex. Consider the plots shown in Figure 6. Five different aspects of satellite's spatial variability are shown:

a. Assuming a known height for the satellite, a single ship just entering the field of view could be seen from an infinite number of possible satellite positions all a radius away from the ship's location.

b. If height is not constrained, clearly there are an infinite number of different sized footprint radii that could see the ship, similar to the illustration in Figure 5 for the 2-D case.

c. Once a second ship can be seen, the infinite space of possible fields of view will shrink, as the fact that the second point is not seen prior to its first entry into the FoV would constrain the possible fields of view prior to its detection.

d. A ship that is seen for maximum length of time amongst all other ships could at best be assumed to have traversed through the FoV diameter, giving a lower bound for satellite height and some constraint on directionality.

e. Visibility of new ships at subsequent epochs would constrain the satellite's likely direction of travel.

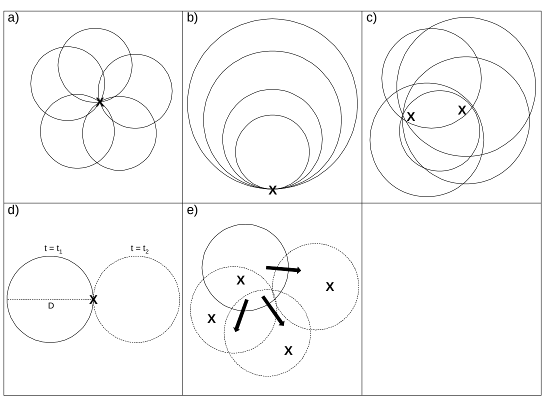

Figure 6: Satellite's Spatial Variability in 2-D

According to the above description, a vast search space for likely field of view over time exists, which directly translates into computational complexity. As such, additional constraints need to be imposed based on domain prior knowledge of the scenario in order to render the problem solvable and ensure numerical stability [10].

Some of the spatial constraints include having knowledge of the range of feasible heights for a given satellite or confining the satellite's initial estimated trajectory to an error bound around, for example, a NORAD TLE propagation. In the temporal domain, satellite motion is able to be constrained quite precisely in the absence of any propulsive manoeuvres, with orbital perturbations being of limited variation and consequently able to be assumed approximately constant over the duration of a region overflight. Current visibility knowledge can be back-propagated to confine the possible location of the satellite as predicted by new ships entering the field of view. Figure 7 shows a conceptual plot of possible satellite locations over time for a fixed height, based on individual ship visibility durations, with the overlap between these at any instant in time being the most likely location for the satellite at that time [9].

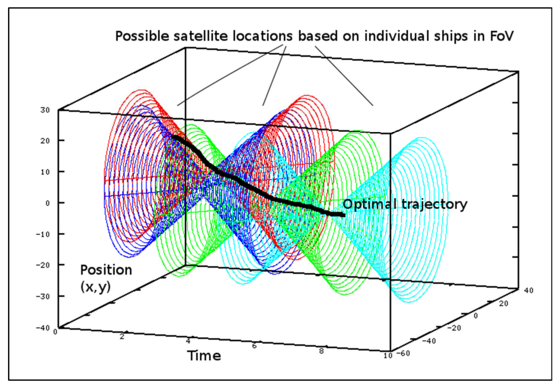

Figure 7: Conceptual 3-D Plot of Possible Satellite Locations

The 3-D conic shapes in Figure 7 represent all possible locations for the satellite at an instant in time, based on knowledge of the ship location and time of seeing the ship, and in the absence of any uncertainty or errors. When the ship first comes into view, the satellite could be at any point where that ship would fall on the edge of its field of view, which for a fixed satellite height reduces approximately to a circle of possible locations (with geoid shape, antenna height etc. deforming the possible locations somewhat from a circle). If the satellite flies directly over the ship, this would be known from the fact that the ship would be in view for the longest possible time for a point seen from the given satellite altitude. Consequently, the "exact" satellite location would be known half-way through the overflight, where it would be directly above the ship. The conic shape therefore reflects the fact that as a ship remains in view for a certain amount of time, the possible satellite locations will converge then diverge during the access. If ships are located off the sub-satellite track, the point of closest approach to the ship will have a circular uncertainty of radius equal to the distance between the subsatellite point of closest approach and the ship. With a large number of ships in view, the overlap of these conic sections provides the most likely satellite position at any time.

Considering the effect of unconstraining satellite height and the error sources in the ship information and time of visibility, the crisp conic sections become probability density functions spread from the crisp section. Combining the probabilities from each ship in the full dataset would provide a most likely overlap trajectory through all these volumes. Determining this most likely trajectory can be considered as a problem of finding the maximal trajectory through the GHT histogram. The location estimates would be obtained from the local maxima.

Consider a single time slice, and three ships in the field of view, where each has been in view for different lengths of time. The probabilistic contributions from all visible ships would be plotted, as shown in Figure 8. From this combined plot, the uncertainty from each ship measurement may be large, but the overlap region would be relatively small, and would be possible to impose higher constrains on the satellite's possible location as more ships are in view.

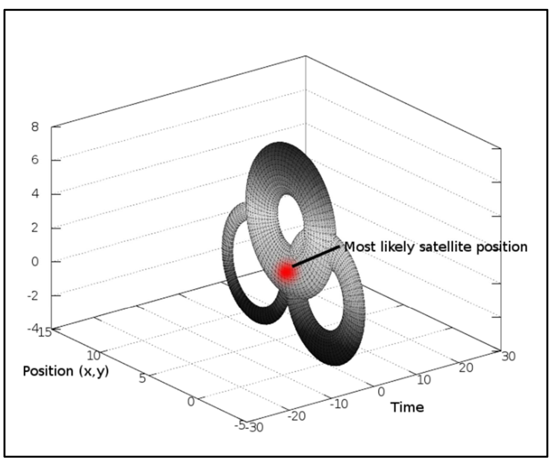

Figure 8: Possible satellite locations derivable from three ships in different locations having been in the field of view for different durations, with different levels of uncertainty.

To use this method in near-real-time, there are two possible implementations, with or without time lag on each measurement: to have the best knowledge of the satellite position with time, we would need to know the time a ship enters and exists the satellite's field of view to best constrain the likely disc of satellite locations for a given timestep. To elaborate, when a ship first comes into view, we know that the satellite is a half-field-of-view away from that ship. If we step in time and know the ship is still in view, we may be a timestep closer to the ship, or alternatively we may be about to lose the ship from the field of view. The closest we could be to the ship would be if we assumed our trajectory was directly towards it, giving a bound to the inner radius of the uncertainty disc. The furthest we could be from the ship is a half-field of view, assuming in the next timestep we would lose the ship from sight, giving an upper bound to the uncertainty radius.

A more constrained set of uncertainty discs could be generated if we processed ship information with some time lag and used the resulting knowledge of times of ships exiting the field of view. Considering the case discussed above, if in the second timestep after having seen a given ship, we know that we will continue to see the ship for a further N timesteps, we know that we will not lose the ship from sight in the very next step and so can more tightly constrain the uncertainty disc's outer bound. This would consequently converge to a solution much faster.

Mapping this to a GHT would involve quantizing the possible satellite location parameter space, and for each position point at time t, determining the number of "hits" (ships that would be able to be in the field of view if the satellite were in the given location, which the measurements report *were* in view) versus "misses" (two possible miss types). The two types of "miss" are:

1. Ships that we know are in view which could not be from the location in the GHT histogram (which would be a very strong indicator of an incorrect satellite location)

2. Ships for which we do *not* have a measurement at the given timestep but which *should* have been seen per the GHT histogram location (these would be a weaker indication of an incorrect location as "missed detections" and

other detection issues could result in these "false negatives")

Thereafter, motion constraints would be used to effectively apply a temporal weighting to each time-step's histogram to weight more likely dynamics solutions, and the most likely location for the satellite at a given timestep would then consist of the peak of the GHT histogram [4].

Some of the advantages of this method include its robustness to partial or erroneous detections which would effectively deform the "detected field of view shape", and its robustness to the presence of other error sources.

IV. The Estimation Problem

A Kalman filter at its simplest is an optimal weighted averaging mechanism between predicted and measured states, with weighting based on uncertainty. Breaking the filter down into predictive and update actions, the predictive element is able to use a motion model with whatever level of model precision desired, subject to sufficient observability being available in the measurements. The update is intended to adjust the prediction sufficiently to track the underlying motion trends without tracking the noise in the measurements [5].

The state being predicted is some representation of the six orbital elements at every timestep and the ships` navigation data and times seen by the spacecraft are the raw measurements of the problem. There are direct and indirect functional relationships between the states and measurements. Thus, the measurements could ultimately be defined as a function of the states, which forms the basis of the filter. It is planned to use an extended or other variant Kalman filter because of the non-linear nature of the functional models [6][7].

The raw ship messages are input into the GHT estimator to obtain the best estimate of the satellite's state vector at t_0. At the next epoch (t_1), the state vector is determined as a function of the state transition model applied to the state at the last epoch, the control input model applied to the control state, plus a noise model.

The Kalman filter therefore needs a functional model, where the measurements (Z) and the states (X) are defined. If the measurements are definable as functions of the states ($Z = f(X)$), then the measurement matrix (H) can be obtained by partially differentiating each measurement with respect to the states. Figure 9 illustrates a standard information flow for a Kalman filter.

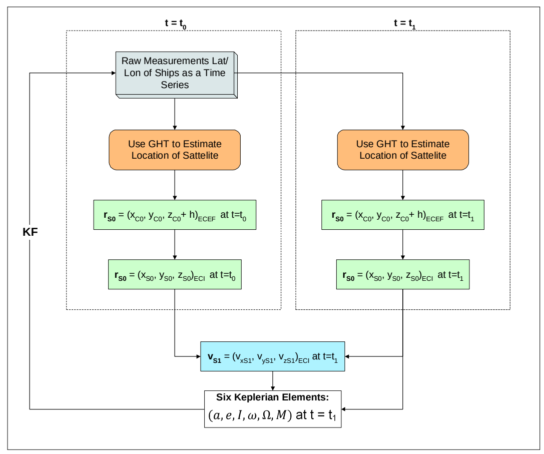

Figure 9: Process Flow of Satellite Orbit Determination Algorithm using Ships' Navigation Data

For the orbit determination problem presented here, a significant advantage of a Kalman filter is the ability for the filter to accept measurements non-uniformly, with the filter propagating its state in the absence of measurements, or making many updates based on many measurements in a timestep, as required.

The problem of defining the functional mapping of the measurements to the states is consequently the key to an efficient and convergent filter solution. The initial approach to be used for this mapping, determining most likely positions of the spacecraft at each epoch, constraining the search space for the next (or previous, in the case of smoothing) epoch based on feasible orbital dynamics, and not mapping measurements directly to a velocity component, does not make optimal use of all the information in the measurements, but provides a first approach for assessing the feasibility of the technique. In future, it is intended to upgrade this mapping, most likely to a direct measurement to

Keplerian element map, but this is the subject of future development once the measurements themselves are better understood.

V. Simulation, Filter Implementation, Optimization and Post-processing

Whilst real-world data exist, it is planned to build up the analysis of the method from initial simplified models in a simulation environment, with the simulation fidelity progressively improved as the orbit determine method matures, to support evolution to a point where real ship data will be able to be used. To simulate the problem, a scenario has been built in STK (Satellite Toolkit by AGI) and calling on appropriate functions from MATLAB to extract access times for regularly spaced grid points. These grid points are stationary, which for now can represent ship locations because the relative velocity of the ships is negligible for the most part compared to the relative velocity between the satellite and the ships. These simulated measurements consist of geodetic latitude and longitude for each of the ships transmitting AIS messages (assumed continuous for now). Therefore, each ship ID will have a corresponding location and a time stamp, which would basically represent a time series of ship locations over a given scenario. Assuming error-free measurements in simulation mode, a point will only appear in the FoV if it is seen by the satellite [7].

There are several tools available to process the heterogeneous measurement data to uniquely solve for the satellite orbit. The complexity lies in mapping and performing filtering on the Hough space to produce the most likely trajectory and its corresponding error covariances. It is currently planned to develop tools mapping the GHT output onto standard input types supported by the industry standard Orbit Determination Toolkit (ODTK) from analytical graphics. This has built-in modeling for many of the perturbations and ability to solve for measurement biases, possible satellite manoeuvres, etc. ODTK has built-in readers for standard data-types that are limited in selection, and so most of the data interfaces will have to be custom defined [8].

Further enhancement of the estimation could be achieved by using the smoothing capabilities of ODTK. In particular, a Variable Lag Smother (VLS) could be of interest for this problem. Two types of formulations are used to implement the VLS: Frazer and Carlton-Rauch. The main advantage of using this filter is that smoothing lag can be varied for fixed epochs. Other statistical constraints can be set to fine tune the desirable outcome, such as covariance reduction completion criteria, number of consecutive accepted measurements and minimum output spacing.

VI. Summary and Conclusions

A new type of ground-based measurement source is being proposed herein to estimate the position of a satellite based on times of visibility of large numbers of known points in the field of view. This holds potential for orbit determination for low-cost missions or for direct computation from associated mission sensor data. In the proposed method, the data is readily available for input – it's only a matter of proper integration and processing of the measurement feed.

The concept proposed in this paper is currently in development and is the subject of an ongoing Ph.D. The paper has presented the ideas behind the proposed approach, and defines some initial simplified formulations and the complexities and robustness issues to be addressed as the approach matures. It is expected that a first full implementation will be developed by early next year.

This initial orbit determination problem may also lead to a host of new applications relating to bathymetry, atmospheric propagation measurement science using measurement regional biases etc.

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