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## computational mathematics



**PRESENTED BY THE WATERLOO INSTITUTE FOR SUSTAINABLE ENERGY AND THE CENTRE FOR COMPUTATIONAL MATHEMATICS IN INDUSTRY AND COMMERCE**

**Thursday, March 26, 2015 2:30 – 3:30 pm CPH 4333**

## **MODELING MULTI-SCALE PROCESSES IN HYDRAULIC FRACTURE PROPAGATION USING THE IMPLICIT LEVEL SET ALGORITHM (ILSA)**

## **Dr. Anthony Peirce**, Professor, Department of Mathematics The University of British Columbia

In this talk I describe an implicit level set algorithm (ILSA) suitable for modeling multi-scale behavior in planar hydraulic fractures propagating in three dimensional elastic media. The novel ILSA scheme (Peirce 2015) is able to represent the required multi-scale behavior on a relatively coarse rectangular mesh. This is achieved by using the local front velocity to construct, for each point of a set of control points, a mapping that adaptively identifies the dominant length scale at which the appropriate multi-scale universal asymptotic solution needs to be sampled. Finer-scale behavior is captured in a weak sense by integrating the universal asymptotic solution for the fracture width over partially filled tip elements and using these integrals to set the average values of the widths in all tip elements. The ILSA solution shows good agreement with a multi-scale reference solution comprising a radial solution that transitions from viscosity to toughness dominated propagation regimes. The ILSA scheme is also used to model blade-like hydraulic fractures that break through stress barriers located symmetrically with respect to the injection point. For the zero toughness case, the ILSA solution shows close agreement to experimental results. The multi-scale ILSA scheme is also used to provide results when the material toughness KIc is non-zero. In this case, different parts of the fracturefree-boundary can be propagating in different regimes. I demonstrate how we have used these reference solutions to construct reduced order models that can execute in a fraction of the original computational time. I also provide examples in which this methodology is used to model multiple hydraulic fractures that propagate simultaneously in parallel planes. These multi-fracture models highlight surprising dynamics between the interacting fractures that indicate significant potential for using numerical design to improve production.

## **Biography**



**Dr. Anthony Peirce**

Anthony Peirce was a Fulbright Scholar at Princeton University, where he received his PhD in Applied and Computational Mathematics in 1987. Prior to his PhD, he worked as an Applied Mathematician at the Chamber of Mines Research Laboratories in South Africa, where he investigated rock fracture processes around underground excavations. His research interests include: the application of control to molecular systems, the analysis of instabilities in elasto-plastic materials, the development of specialized numerical algorithms to model largescale rock fracture processes, numerical and analytic studies of reactive flows in porous media, and more recently, the asymptotic and numerical analysis of fluid-driven-fracture propagation. Further details are available on his website: www.math.ubc.ca/~peirce

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