New Real-Time Demand Response Market Co-Optimized With Conventional Energy Market

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***Abstract*—In addition to procuring energy, consumers in electricity markets procure demand response (DR) services. Demand and supply of energy in the electricity market drives the demand for DR services. Through the net benefits test (NBT), economic procurement of DR is limited to an amount that ensures that consumers benefit with the procurement of DR services. However, the NBT neither a) recognizes the coexistence of the DR market with the energy market; nor b) optimizes social welfare in the DR market in concert with that of the energy market. This lack of accounting for DR market surplus results in economic inefficiency. To address this shortcoming, we advance past works by: a) proposing a real-time DR market where the DR demand curve is a function of opportunity in the energy market; and b) co-optimizing energy and DR markets such that the total social welfare derived from both markets is maximized simultaneously. We also present an optimal power flow formulation and process to implement our ideas in real-time electricity markets. The formulation is tested on a simple test case and a system based on actual Pennsylvania-New Jersey-Maryland (PJM) data. For the PJM case, total social welfare is increased by 1.41% to 3.05% over existing DR procurement strategies, resulting in \$14.5M to \$30.9M additional benefits per hour.**

***Index Terms*—Demand response, optimal power flow, power system economics.**

NOMENCLATURE

Indices

$i,\ j$	Bus indices.
k	Line index.
m, n	Indices for demand response supply and demand.

Parameters

NB	Total number of buses.
NPRS, NPRD	Number of demand response supply and demand units.
$PD_i,\ QD_i$	Real and reactive power consumed at bus i.
PD_T	Total real power consumed in system.
$Y_{ij}\angle\theta_{ij}$	Bus admittance matrix element between buses i and j.
$y_{ij}\angle\phi_{ij}$	Line admittance element between buses i and j.
$\underline{V_i},\ \overline{V_i}$	Minimum and maximum voltage limits at bus i.
$\overline{SL_k}$	Maximum apparent power flow possible in line k.
NT	Total number of transmission lines and transformers.
$\underline{PG_i},\ \overline{PG_i}$	Minimum and maximum real power generation limits at bus i.
$\underline{QG_i},\ \overline{QG_i}$	Minimum and maximum reactive power generation limits at bus i.
$\overline{PR_i}$	Maximum demand response limit at bus i.
a, b, c, d	Constants in generator's price offer curve.

Variables

$PG_i,\ QG_i$	Real and reactive power generated at bus i.
PG_T	Total real power generated in system.
$PR_i,\ PR_T$	Total demand response dispatched at bus i and in entire system.
$PR_{S,m}, PR_{D,n}$	Demand response quantity for supply unit m and demand unit n.
$V_i\angle\delta_i$	Voltage phasor at bus i.
$SF_k,\ ST_k$	Apparent power flow from and to line k, respectively.
$\lambda 0_i^E$	Locational marginal price at bus i , without DR.
λN_i^E	Locational marginal price at bus i , with DR.
λ^{DR}	Price paid for DR.

Other

NtB	Net benefits.
BBR	Buyers' benefit due to demand response.
BCR	Buyers' cost due to demand response.
$F_S^E(PG_T)$	Energy supply cost curve as a function of PG_T.
$\lambda_S^E(PG_T)$	Energy supply price curve as a function of PG_T.
$\lambda_D^E(PG_T)$	Energy demand price curve as a function of PG_T.
$\lambda_S^{\mathrm{DR}}(PR_T)$	Demand response supply price curve as a function of PR_T.
$\lambda_D^{\mathrm{DR}}(PR_T)$	Demand response demand price curve as a function of PR_T.
ECS	Energy consumers' surplus.
ESS	Energy suppliers' surplus
DRCS	Demand response consumers' surplus.
DRSS	Demand response suppliers' surplus.

I. INTRODUCTION

DEMAND response (DR) is a unique resource. It has many applications and reasons for procurement. While much

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of DR procurement is for reliability purposes, some of it is purchased to drive economic efficiency. Some early published literature appeared in the 1970s as load management [1], [2].

However, DR's full economic impact must be well-understood to ensure optimal market operations and DR procurement practices that treat all market participants fairly.

A. Literature Review

DR is "changes in electric usage by demand-side resources from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized" [3]. This expansive definition covers a wide range of DR categories, as shown in Fig. 1. In the broadest sense, DR can be price-based, which changes consumption based on price signals, or incentive-based, which is incentivized financially [4]–[7]. Incentive-based DR can be classical or market-based, which in turn can be paid for capacity, energy, or other schemes. This article explores *market-driven incentive-based DR programs in which DR suppliers are compensated for their services through demand bidding*. DR procurement for other purposes such as reliability or reserves is out of scope for this article. Furthermore, existing and established baseline practices (such as [8]–[13]) are used to evaluate, measure, and verify DR load reductions.

Within this category of demand bidding, market-based, incentive-based DR, there are three types of market participants: 1) generators; 2) loads; and 3) DR. Fig. 2(a) shows a schematic of a simple illustrative system with no DR. In this case, there is only one type of vendor—the generators—and one type of consumer—the loads. However, when DR is introduced, as in Fig. 2(b), a subset of the loads (consumers) transform into DR (vendors). There are now two vendors—generators and DR—while loads are the consumers. The remaining pool of consumers is smaller, and the most expensive generator will be displaced by the DR [14].

DR functions differently than a generator. This dilemma was discussed and debated extensively, particularly in the consultations leading to Federal Energy Regulatory Commission (FERC) Order 745, which was issued in 2011 [15]. FERC, along with numerous stakeholders, identified the *billing unit effect* due to incentive-based, market-based, demand bidding DR. Specifically, generators increasing production is fundamentally different than increasing DR because DR involves loads leaving the consumer pool. This means the total costs are borne by a smaller pool of remaining customers, thereby driving up actual prices [14]. Therefore, generation and DR affect the market very differently, and they must be treated differently to avoid unintended distortions.

A comparison of approaches in this category of market-based, incentive-based DR is summarized in Table I, and each approach will be described in more detail next. The last row describes the proposed method and compares it with published methods. For reserve markets in practice, DR is primarily procured through capacity markets to provide operating reserve services [16], [17]. This is done months in advance of the need, and the demand curve for DR is based on the cost of new entry. For reserve markets in literature, co-optimization of energy and DR in reserve markets in real time has been explored. This is done through the

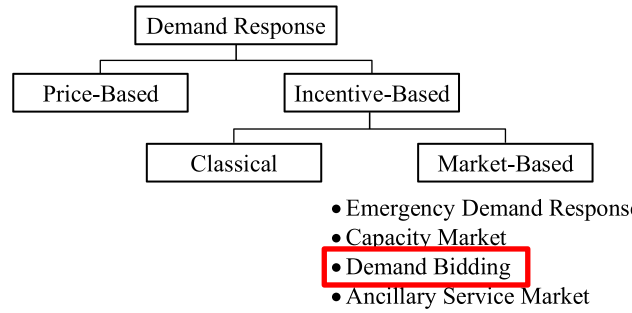
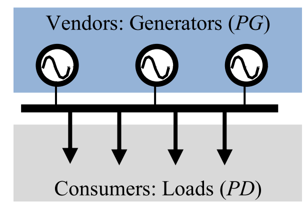
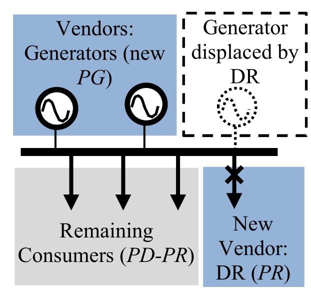


Fig. 1. Demand response categories and subcategories.



(a)

Fig. 2(a). Schematic diagram of simple system without demand response.



(b)

Fig. 2(b). Schematic diagram of simple system with demand response.

TABLE I
COMPARISON OF MARKET-BASED DEMAND RESPONSE APPROACHES

Purpose	Time	Method	Basis of DR Demand Curve	Impact of DR on Energy Market	References
Operating reserve service	Months ahead	Capacity market	Cost of new entry	Not considered	[16], [17]
Operating reserve service	Real-time	DR & generation costs treated the same	None	Not considered	[18]–[21]
Core energy demand	Real-time	DR & generation costs treated the same	None	Not considered	[22]–[26]
Core energy demand	Real-time	Settle Demand Response Exchange (DRX) first; then settle energy market	Bids	Not considered	[27]–[29]
Core energy demand	Real-time	DR quantity limited by NBT	None	Energy market protected by NBT	[15], [30], [31]
Core energy demand	Real-time	Settle DR and energy markets simultaneously	Energy market	Captured through DR demand curve	Proposed in this paper

minimization of DR ancillary service costs alongside generation costs [18]–[21]. However, these all overlook the social welfare and surpluses involved in DR procurement as well as the impact of DR on the energy market. For meeting core energy demands, many demand-bidding DR implementations in academic literature minimize total costs, typically including generation and DR costs [22]–[26]. This takes a traditional generation-only electricity market and adds DR without considering the full economic implications. In a single-ended auction with a single commodity—generation—minimizing total costs is the same as maximizing social welfare. However, with DR there are now two commodities being procured simultaneously, and we will show that the previous practice of minimizing total costs no longer automatically maximizes social welfare because of DR's unique characteristics in reducing the buyers' pool. Demand response exchanges (DRX) have been proposed [27]–[29], however, their impact on conventional energy markets has not been explored. Pool-based DRX typically solicit bids from buyers, which can be transmission system operators, retailers, or distributors. With the exception of [32], past papers do not say how these bid offers are determined on behalf of buyers, a shortcoming they acknowledge. In this article, we will determine the demand curve for DR based on energy market characteristics, thereby articulating and respecting the link between the energy and DR markets.

The US Federal Energy Regulatory Commission (FERC) issued Order 745 in 2011 creating a framework, the net benefits test (NBT), that ensures that buyers of electricity benefit from the procurement of DR services [15]. While the NBT by itself ensures that customers are protected, it does not recognize that the DR market has social welfare, which can be maximized alongside optimization of the energy market. This article proposes this co-optimization of energy and DR markets simultaneously.

This literature survey clearly shows that there is no method that procures DR in real-time for core energy demand purposes through the simultaneous settlement of DR and energy markets while also capturing the impact of DR on the energy market. Furthermore, it must be pointed out that a plethora of research has been completed such as [36]–[38]. These works point to use of specific DR resources, such as residential resources. As an outcome of this paper's method, DR services from these sources may procured when economic.

B. Research Gap

From the preceding literature survey, it is evident that a large amount of work on DR has been completed. However, one fundamental aspect remains unaddressed. A DR market, definition of its social welfare and co-optimization of this DR market's social welfare with social welfare of energy market has never been done. For this reason, while energy market is co-optimized with DR, DR market's social welfare has not been maximized and, hence, the net social welfare is not the most.

C. Main Contributions of This Article and Advantages

The main contributions of this article include the following.

1) *Real-time DR market:* We create a real time DR market where the DR demand curve is derived by maximizing the net benefit of customers of the energy market, considering energy, and DR services.

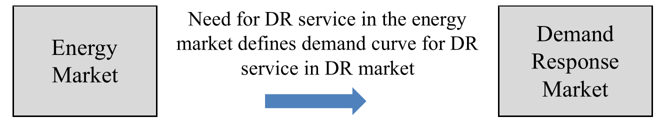


Fig. 3. Dependency of the demand response market on the energy market.

2) *Combined visualizations of energy and DR markets:* We create combined graphs that show activity and social welfare from the energy and DR markets, helping to visualize the relationship and co-optimization of the two markets in maximizing total social welfare.
3) *Co-optimization of dual markets:* We show how to co-optimize energy and DR markets simultaneously by maximizing the total social welfare of the markets. We present an optimal power flow (OPF) formulation to apply our ideas in a real-time market. We demonstrate benefits via study results on a constructed simple test case and real Pennsylvania-New Jersey-Maryland (PJM) system data.
4) *Advantage:* This method enables simultaneous optimization of DR market with energy market, ensuring that all participants, i.e., buyers and sellers of energy and DR services collectively maximize their benefits.
5) *Advantage:* The proposed method ensures that DR procurement—for economic purposes—is limited to when social welfare is maximized, shielding customers from harm.

II. Energy Market Model Considering DR

When an electricity system simultaneously trades in both energy and DR to satisfy demand, there are two separate but interrelated markets—the energy market and the DR market—to trade in the two commodities, energy, and DR. The two markets are related by the fact that total energy demand equals energy traded in the energy market and DR services traded in the DR market. Furthermore, the demand for DR is created from the energy market. While the energy market can exist on its own, the DR market is dependent on the energy market for its existence. That dependancy exists via the creation of a DR market demand curve by the need shown in the energy market, as shown in Fig. 3. We present these details in this and the following section.

A. Supply and Demand in Energy Market

DR affects the energy market very differently than other resources. The energy market settlement graph for a single-ended auction in Fig. 4 shows a supply price curve $\lambda_S^E(PG_T)$ fitted from generation offers stacked from lowest to highest prices. It is a function of PG_T , the total quantity of generation dispatched. For a single-ended auction, demand is inelastic and is represented as a vertical line at total load PD_T and a horizontal line for $0 < PG_T < PD_T$ at a high price λD^E . In the absence of DR, the settlement price is $\lambda 0^E = \lambda_S^E\,(PD_T)$. Furthermore, the supply cost curve $F_S^E(PG_T)$ and supply price curve $\lambda_S^E(PG_T)$ are related as follows:

$$F_S^E\ (PG_T = \text{Limit}) = \int_0^{\text{Limit}} \lambda_S^E\,(PG_T)\ dPG_T \qquad (1)$$

where $\left[PG_T = \sum_{i=1}^{NB} PG_i\right]$ and $\left[PD_T = \sum_{i=1}^{NB} PD_i\right]$.

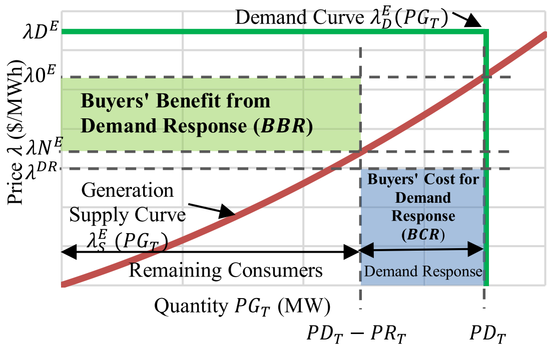


Fig. 4. Energy market settlement graph with demand response procurement, showing buyers' benefit and cost for demand response.

B. Impact on Energy Market Due to Demand Response

When total DR of quantity PR_T is purchased, several noteworthy impacts happen to the energy market.

First, DR reduces the pool of paying consumers. A portion of the consumers (quantity PR_T) convert from being loads to becoming DR service providers. Since these former loads are no longer consuming, they are also no longer paying for energy. This results in a smaller paying pool of consumers of quantity $PD_T - PR_T$, where $PR_T = \sum_{i=1}^{NB} PR_i$.

Second, the most expensive generator units are displaced, thereby decreasing the energy market clearing price. Total generation quantity PG_T reduces from PD_T to $PD_T - PR_T$, and the corresponding energy price drops from $\lambda 0^E = \lambda_S^E\ (PD_T)$ to $\lambda N^E = \lambda_S^E\ (PD_T - PR_T)$.

Third, buyers' benefit increases by BBR , as shown in Fig. 4. This is the new additional consumers' surplus that is gained through the purchase of DR and is defined as

$$\text{BBR} = \left(\lambda 0^E - \lambda N^E\right) \cdot (PD_T - PR_T). \quad (2)$$

Fourth, a new cost is incurred for the purchase of the DR, and that is shown as BCR in Fig. 4. This new cost is defined in (3) and is the quantity of DR purchased (PR_T) multiplied by the price paid for DR (λ^{DR}). Our method produces a market-based price for DR by co-optimizing the energy and DR markets, and this will be described later in the article

$$\text{BCR} = \lambda^{\text{DR}} \cdot PR_T. \quad (3)$$

C. Net Benefits Due to Demand Response

The concept of net benefits is necessary to relate energy and DR markets. In Order 745, the FERC stipulated that net benefits gained through DR should exceed net costs incurred due to DR [15]. Therefore, the net benefits (NtB) must be greater than zero. Net benefit is defined using (2), (3), and $PG_T = PD_T - PR_T$ in (4), and it is visualized in Fig. 4

$$\begin{aligned} NtB &= \text{BBR} - \text{BCR} \\ &= \left(\lambda 0^E - \lambda N^E\right) \cdot PG_T - \lambda^{\text{DR}} \cdot PR_T. \end{aligned} \quad (4)$$

The net benefit per unit of electricity enjoyed by paying consumers $PD_T - PR_T$ then becomes

$$\frac{NtB}{PD_T - PR_T} = \frac{\left(\lambda 0^E - \lambda N^E\right) \cdot (PD_T - PR_T) - \lambda^{\text{DR}} \cdot PR_T}{PD_T - PR_T}. \quad (5)$$

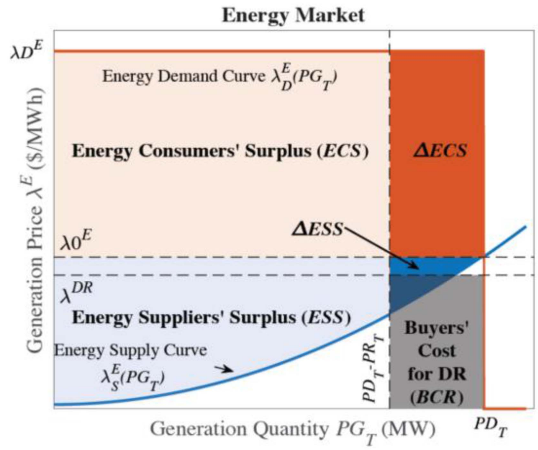


Fig. 5. Energy market settlement graph with demand response, showing change in social welfare due to demand response.

Given that $\lambda 0^E = \lambda_S^E(PD_T)$, $\lambda N^E = \lambda_S^E(PD_T - PR_T)$, and $PG_T = PD_T - PR_T$, (5) can be simplified as a function of PG_T

$$\begin{aligned} &\frac{NtB}{PG_T} \\ &= \frac{\left[\left(\lambda_S^E\ (PD_T) - \lambda_S^E\ (PG_T)\right) \cdot PG_T - \lambda^{\text{DR}} \cdot (PD_T - PG_T)\right]}{PGT} \\ &= \lambda_S^E\ (PD_T) - \lambda_S^E\ (PG_T) - \frac{\lambda^{\text{DR}} \cdot (PD_T - PG_T)}{PG_T}. \end{aligned} \quad (6)$$

For a given price of DR (λ^{DR}) , the best economic strategy for the purchase of DR would be to determine an optimal quantity of (PR_T) that ensures that net benefits BBR − BCR are maximized such that electricity market consumers are best served. This strategy helps determine the demand curve for DR in a real-time DR market, as shown later in Section III-B.

D. Effect on Social Welfare in Energy Market

The procurement of DR alters the social welfare produced in the energy market. Social welfare of a market "is defined as the summation of consumer surplus and supplier surplus" [33]. It is the collective benefit amassed by all market participants, and it provides a foundation for our proposed method.

An energy market with DR is shown in Fig. 5. The energy consumers' surplus (ECS) is the area bound by the demand curve $\lambda_D^E(PG_T)$, the horizontal line for energy market clearing price of $\lambda 0^E$ and the vertical axis. The energy suppliers' surplus (ESS) is the area bound by the supply curve $\lambda_S^E(PG_T)$, the horizontal line for energy market clearing price of $\lambda 0^E$, and the vertical axis. Together, the sum of ECS and ESS constitutes the total social welfare in the energy market.

The procurement of DR has important consequences for social welfare in the energy market. As demand reduces from PD_T to $PD_T - PR_T$, the portion of the energy market formerly covered by PR_T no longer exists; it will transfer to the DR market, which will be described in the following section. Social welfare is, therefore, lost, as shown in the darkened areas in Fig. 5. Therefore, consumers' surplus is reduced by ΔECS , and suppliers' surplus is reduced by ΔESS . Furthermore, energy

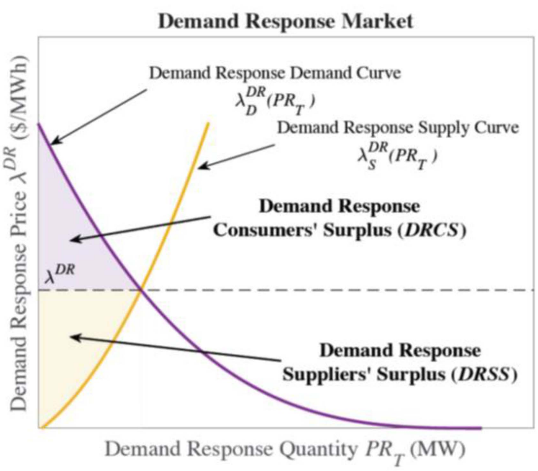


Fig. 6. Demand response market settlement graph showing social welfare.

market consumers must incur a new, additional cost to buy DR, BCR, which is superimposed on Fig. 5. BCR is the investment from the energy market to the DR market. In other words, it is the payments that energy market consumers must make in order to obtain DR.

Therefore, it is a net cost, loss, or investment from the energy market's perspective. BCR is the product of the DR quantity PR_T and the DR price λ^{DR}. Hence, social welfare in the energy market is reduced by $\Delta\mathrm{ECS} + \Delta\mathrm{ESS} + \mathrm{BCR}$ when DR is purchased. Accordingly, any reoptimization of the electricity system with DR must minimize

$$\begin{aligned}&[\text{Social Welfare Lost in Energy Market}] +\\&[\text{Investment from Energy Market into DR market}]\\=\ &[\Delta\mathrm{ECS} + \Delta\mathrm{ESS}] + [\mathrm{BCR}]. \end{aligned} \tag{7}$$

III. Creation of Real-Time DR Market

The DR market comprises supply from DR service providers and demand from remaining electricity consumers, as shown in Fig. 6 [32]. While [32] provided the necessary background, a new, real-time DR service market is proposed in this article. A DR market settlement graph is shown in Fig. 6, and its key elements will be discussed next.

A. Supply in Demand Response Market

In a real-time DR market, interested suppliers of DR service can submit offers, similar to how generators do today. These offers can then be stacked from lowest to highest price to form the DR supply curve $\lambda_S^{\mathrm{DR}}(PR_T)$, which is a function of PR_T, the total quantity of DR purchased, as shown in Fig. 6.

B. Demand in Demand Response Market

The DR demand curve is the crucial element that links the energy and DR markets together, and it is depicted in Fig. 6. The consumers in the DR market are the same as the remaining, paying consumers in the energy market after DR is purchased, i.e., $PD_T - PR_T$.

The consumers' appetite for DR depends on DR's ability to maximize the net benefits (6) gained by those consumers. Therefore, consumers' demand for DR, i.e., how much they are willing to pay for DR, is determined by maximizing the net benefits per unit at each quantity of $PG_T = PD_T - PR_T$. This is determined by finding the partial derivative of (6) with respect to PG_T and setting it to zero, as follows:

$$\begin{aligned}\frac{\partial\left(\frac{NtB}{PG_T}\right)}{\partial PG_T} &= \frac{-\partial\,\lambda_S^E\left(PG_T\right)}{\partial\,PG_T}\\&\quad-\frac{\begin{bmatrix}-\lambda_D^{\mathrm{DR}}\left(PG_T\right)\cdot PG_T\\-\lambda_D^{\mathrm{DR}}\left(PG_T\right)\cdot\left(PD_T - PG_T\right)\end{bmatrix}}{{PG_T}^2}\\&= \frac{-\partial\,\lambda_S^E\left(PG_T\right)}{\partial\,PG_T} + \frac{\lambda_D^{\mathrm{DR}}\left(PG_T\right)\cdot PD_T}{{PG_T}^2} = 0.\end{aligned} \tag{8}$$

Rearranging (8), we obtain the demand curve for DR

$$\lambda_D^{\mathrm{DR}}\ \left(PG_T\right) = \frac{\partial\,\lambda_S^E\left(PG_T\right)}{\partial\,PG_T}\cdot\frac{{PG_T}^2}{PD_T}. \tag{9}$$

Since $PG_T = PD_T - PR_T$, the demand curve can be redefined in terms of PR_T, and this is the demand curve for DR $\lambda_D^{\mathrm{DR}}(PR_T)$ plotted in Fig. 6:

$$\lambda_D^{\mathrm{DR}}\left(PR_T\right) = \frac{-\,\partial\,\lambda_S^E\left(PD_T - PR_T\right)}{\partial\,PR_T}\cdot\frac{\left(PD_T - PR_T\right)^2}{PD_T}. \tag{10}$$

Equation (10) provides the relationship between the real-time energy market through λ_S^E and the real-time DR market through λ_D^{DR}. For a given DR price of $\lambda_D^{\mathrm{DR}}(PR_T)$, the optimal quantity of DR PR_T is determined so that the energy market buyers' benefit is maximized. Equation (10) is the demand curve for DR in terms of $\mathrm{PR_T}$, the quantity of DR. It represents the price buyers (from the energy market) are willing to pay at each DR quantity. If the price is higher than the curve, then they would not want to buy DR as that would make them worse off. If the price is lower than the curve, then they would want to buy the DR and enjoy the surplus, which is the difference between the price and the curve.

C. Social Welfare in Demand Response Market

For the first time, the full social welfare in the real-time DR market—a societal gain due to DR—is maximized. The DR consumers' surplus (DRCS) is the shaded area below the DR demand curve $\lambda_D^{\mathrm{DR}}(PR_T)$ and above the DR market clearing price λ^{DR} in Fig. 6. This is the benefit enjoyed by consumers, as they are willing to pay up to $\lambda_D^{\mathrm{DR}}(PR_T)$ but instead are charged the DR market clearing price of λ^{DR}.

The DR suppliers' surplus (DRSS) is the shaded area above the DR supply curve $\lambda_S^{\mathrm{DR}}(PR_T)$ and below the DR market clearing price λ^{DR}. This represents the profits or surplus for DR sellers. Together, DRCS and DRSS form the social welfare in the DR market. Integrating $\lambda_D^{\mathrm{DR}}(PR_T)$ and $\lambda_S^{\mathrm{DR}}(PR_T)$ with respect to PR_T yields the DR demand cost function $F_D^{\mathrm{DR}}(PR_T)$ and the DR supply cost function $F_S^{\mathrm{DR}}(PR_T)$ for PR_T, as follows:

$$F_D^{\mathrm{DR}}\left(PR_T\right) = \int_0^{PR_T}\lambda_D^{\mathrm{DR}}\left(PR_T\right)\,dPR_T \tag{11}$$

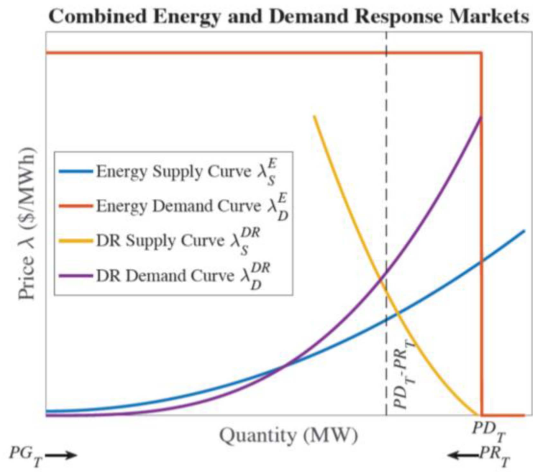


Fig. 7. Combined settlement graph for both energy and demand response markets, with generation quantity PG_T ascending from left to right, and DR quantity PR_T ascending from right to left.

$$F_S^{\mathrm{DR}}(PR_T) = \int_0^{PR_T} \lambda_S^{\mathrm{DR}}(PR_T)\, dPR_T. \quad (12)$$

The social welfare of the DR market can be computed as

$$\mathrm{DRCS} + \mathrm{DRSS} = F_D^{\mathrm{DR}}(PR_T) - F_S^{\mathrm{DR}}(PR_T). \quad (13)$$

We propose to jointly maximize the combined social welfares in real-time energy and DR markets in the following section.

IV. Combined Visualizations of Real-Time Energy Market and Real-Time DR Market

Our proposed method maximizes the combined social welfare in both the energy market and the DR market simultaneously. We also propose a practical formulation to implement our proposed method. The two markets—energy and DR—can be presented in a single chart for easy visualization, as shown in Fig. 7. Since $PD_T = PG_T + PR_T$, the DR market settlement graph in Fig. 6 can be flipped and reflected along its *y*-axis and then superimposed on the energy market graph in Fig. 5 starting at PD_T. PG_T starts at the *y*-axis and ascends moving right, while PR_T starts at PD_T and ascends moving left.

Any particular solution or scenario is represented by a vertical line at $PD_T - PR_T = PG_T$, and a sample is shown in Fig. 7. Some combination of generation PG_T and DR PR_T must be procured to satisfy total load PD_T. The location of this vertical line $PD_T - PR_T = PG_T$ determines the total social welfare derived from the energy and DR markets. Shifting the line $PD_T - PR_T = PG_T$ to the left reduces the social welfare from the energy market and increases the social welfare from the DR market, and vice versa. This relationship between social welfare and PR_T is depicted in Fig. 8. Clearly, there is a point at which the combined total social welfare from the energy and DR markets is at the maximum. This is the optimal PR_T for the system, and we present how to find this in the following section.

The "humps" in Fig. 8 represent the conditions under in which DR can be purchased. On Fig. 6, the DR market settlement graph shows that DR can be purchased from 0 up to the point at which the supply and demand curves intersect. Beyond that, there is no market because the suppliers' desired price is above the consumers' desired price.

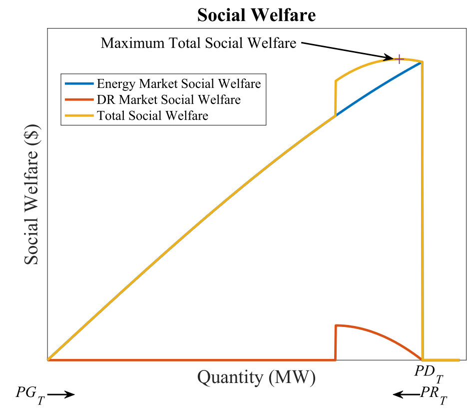


Fig. 8. Social welfare for various procurement combinations of generation and demand response, with generation quantity PG_T ascending from left to right, and DR quantity PR_T ascending from right to left.

V. Proposed Methodology for Co-Optimization

Our proposed method, comprising a practical formulation and implementation process, for co-optimization of energy and DR markets maximizes the combined total social welfare in both the energy market and the DR market simultaneously.

A. New Algorithm: Optimal Power Flow that Maximizes Total Social Welfare of Energy and DR Markets

Our proposed method is a standard optimization problem with an objective function subject to a series of constraints.

1) Objective Function: The objective is to maximize the total combined social welfare in the energy and DR markets. The starting point is the energy market optimized using a conventional OPF, without DR. When DR is introduced, social welfare *decreases* in the energy market and *increases* in the DR market. Therefore, maximizing the total social welfare means simultaneously minimizing the social welfare *lost* in the energy market and maximizing the social welfare *gained* in the DR market. Mathematically, the objective function is

$$\text{Maximize}: \begin{bmatrix}\text{Social welfare (SW)}\\ \text{in energy market}\end{bmatrix} + \begin{bmatrix}\text{Social welfare (SW)}\\ \text{in DR market}\end{bmatrix}.$$

When starting with an optimized energy only market without DR, introduction of DR results in changes in SW for energy and DR markets. Hence, the objective can be reduced to maximizing changes in SW for the two markets

$$\text{Maximize}: \begin{bmatrix}\text{Change in SW}\\ \text{in energy market}\end{bmatrix} + \begin{bmatrix}\text{Change in SW}\\ \text{in DR market}\end{bmatrix}.$$

This objective may be revised and restated as follows:

$$\text{Maximize}: -\begin{bmatrix}\text{SW lost}\\ \text{in energy}\\ \text{market}\end{bmatrix} - \begin{bmatrix}\text{Investment from}\\ \text{energy market}\\ \text{into DR market}\end{bmatrix} + \begin{bmatrix}\text{SW}\\ \text{gained in}\\ \text{DR market}\end{bmatrix}.$$

Mathematically, it is expressed as follows:

$$\text{Maximize}: -[\Delta\text{ECS} + \Delta\text{ESS}] - [\text{BCR}] + [\text{DRCS} + \text{DRSS}]. \quad (14)$$

The first three terms are the changes in the energy market for energy consumers' surplus, energy suppliers' surplus, and buyers' cost for DR, all in (7). The last two terms are the DR consumers' surplus and DR suppliers' surplus, both from (13).

From Fig. 5, the change in energy consumers' surplus ΔECS can be represented as (15), with PR as one of the decision variables, whereas λ_D^E and $\lambda 0^E$ are parameters

$$\Delta\text{ECS} = \sum_{i=1}^{NB} PR_i \cdot \left(\lambda_{Di}^E - \lambda 0_i^E\right). \quad (15)$$

The change in energy suppliers' surplus ΔESS equals

$$\Delta\text{ESS} = \left[\sum_{i=1}^{NB} \lambda 0_i^E \cdot PR_i\right] - \left[F_S^E\left(\sum_{i=1}^{NB} PD_i\right) - F_S^E\left(\sum_{i=1}^{NB} PG_i\right)\right]. \quad (16)$$

The buyers' cost for DR BCR is defined in (3). Since the price paid for DR λ^{DR} is $\lambda_S^{\text{DR}}(\sum_{i=1}^{NB} PR_i)$, BCR is

$$\text{BCR} = \lambda_S^{\text{DR}}\left(\sum_{i=1}^{NB} PR_i\right) \cdot \sum_{i=1}^{NB} PR_i. \quad (17)$$

The DR market surplus is the sum of the DR consumers' surplus DRCS and the DR suppliers' surplus DRSS, as shown in Fig. 6. Mathematically, this is defined in (18) and based on (13)

$$\text{DRCS} + \text{DRSS} = F_D^{\text{DR}}\left(\sum_{i=1}^{NB} PR_{D,i}\right) - F_S^{\text{DR}}\left(\sum_{i=1}^{NB} PR_{S,i}\right). \quad (18)$$

2) Constraints: The objective function (14) is subject to these constraints as follows.

1) *Energy market real power balance:* Real power must be balanced at every bus i, as in (19). Note that PR_i is for DR

$$PG_i + PR_i - PD_i - V_i \sum_{j=1}^{NB} V_j \cdot Y_{ij} \cdot \cos\delta_i - \delta_j - \theta_{ij} = 0 \quad \forall\, i = 1 \; to \; NB. \quad (19)$$

2) *Energy market reactive power balance:* Likewise, reactive power must be balanced at every bus i, as in (20)

$$QG_i - QD_i - V_i \sum_{j=1}^{NB} V_j \cdot Y_{ij} \cdot \sin\delta_i - \delta_j - \theta_{ij} = 0 \quad \forall\, i = 1 \; to \; NB. \quad (20)$$

3) *Demand response market balance:* The supply PR_S and demand PR_D in the DR market must be balanced, as in

$$\left(\sum_{m=1}^{\text{NPRS}} PR_{S,m}\right) - \left(\sum_{n=1}^{\text{NPRD}} PR_{D,n}\right) = 0. \quad (21)$$

4) *Link between energy and DR markets:* The DR quantity at bus i is the sum of the dispatched DR supply units at that bus

$$PR_i = \sum_{n \in i} PR_{S,n} \;\; \forall\, i = 1 \; to \; NB. \quad (22)$$

5) *Voltages limits:* Minimum ($\underline{V_i}$) and maximum ($\overline{V_i}$) voltage limits at every bus i must be respected, as in

$$\underline{V_i} \le V_i \le \overline{V_i} \;\forall\, i = 1 \; to \; NB. \quad (23)$$

6) *Line flow limits:* Line flows from (SF_k) and to (ST_k) every line k are defined in (24) and (25), respectively, and they must respect the maximum limit $\overline{SL_k}$ as defined in (26) and (27) as follows:

$$SF_k = [(V_i\angle\delta_i - V_j\angle\delta_j) \cdot y_{ij}\angle\phi_{ij}]^* \cdot V_i\angle\delta_i \quad \forall\, k = 1 \; to \; NT,\; \{i,j\} \in k \quad (24)$$

$$ST_k = [(V_j\angle\delta_j - V_i\angle\delta_i) \cdot y_{ij}\angle\phi_{ij}]^* \cdot V_j\angle\delta_j \quad \forall\, k = 1 \; to \; NT,\; \{i,j\} \in k \quad (25)$$

$$|SF_k| \le \overline{SL_k} \;\forall\, k = 1 \; to \; NT \quad (26)$$

$$|ST_k| \le \overline{SL_k} \;\forall\, k = 1 \; to \; NT. \quad (27)$$

7) *Generator real and reactive power limits:* The generators must operate within their real and reactive power operating limits. The real and reactive power (PG_i, QG_i) at every bus i must be between the minimum ($\underline{PG_i}$, $\underline{QG_i}$) and the maximum ($\overline{PG_i}$, $\overline{QG_i}$) limits, as in

$$\underline{PG_i} \le PG_i \le \overline{PG_i} \;\forall\, i = 1 \; to \; NB \quad (28)$$

$$\underline{QG_i} \le QG_i \le \overline{QG_i} \;\forall\, i = 1 \; to \; NB. \quad (29)$$

8) *Demand response limits:* Finally, the DR PR_i dispatched at every bus i must be less than the maximum limit $\overline{PR_i}$ and not be negative, as in

$$0 \le PR_i \le \overline{PR_i} \forall i = 1 to NB. \quad (30)$$

This formulation (14) to (30) is an optimization challenge and can be solved with any classical optimization technique. A nonlinear formulation of the OPF is chosen because it is more accurate and realistic than a linear approximation as it accounts for losses. While a nonlinear formulation is chosen, it could easily be adapted to a linear, dc approximation as power balance equations. A generic formulation has been provided here to demonstrate the core contribution of maximizing total energy and DR social welfare, with the intention that it could be adapted to specific user circumstances.

B. Implementation Process

To solve the formulation (14) to (30), a new preprocessing step is required in order to obtain the complete solution, as shown in Fig. 9.

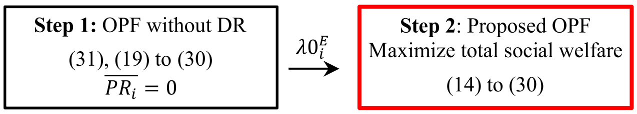


Fig. 9. Implementation process.

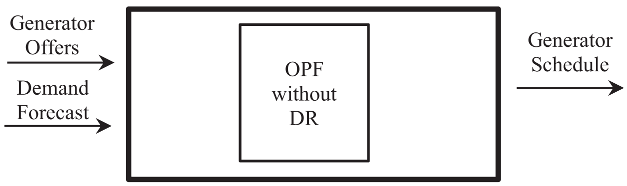


Fig. 10. Existing real-time settlement process.

Step 1: Beforehand, it is necessary to solve a regular OPF without DR in order to obtain $\lambda 0^E$ at every bus for use in (15) and (16). The objective function is to maximize social welfare

$$\text{Maximize} \sum_{i=1}^{NB} PD_i \cdot \lambda_{Di}^E - F_S^E\left(\sum_{i=1}^{NB} PG_i\right). \tag{31}$$

The objective is subject to constraints (19) to (30), which are the same as previously, with the DR limit (30) set to $\overline{PR_i} = 0$, which forces the DR at every bus to be zero. Note that Step 1 is simply a preprocessing step; its purpose is solely to obtain the $\lambda 0^E$ values, which then become inputs into Step 2 in (15) and (16).

Step 2: Solve the OPF proposed previously in (14)–(30), maximizing total social welfare of the energy and DR markets. The energy and DR markets are co-optimized in this step.

It is not possible to consolidate this into a single step. This is because $\lambda 0^E$ is required at each node in the objective function of the co-optimized formulation in (15) and (16). The only way to obtain the $\lambda 0^E$ values is through a preprocessing step.

In Step 1, the locational marginal prices are the Lagrangian multipliers corresponding to the real power balance constraints. The objective function in Step 1 is to minimize total generation costs. In Step 2, the objective function becomes maximizing total social welfare. Therefore, the Lagrangian multipliers no longer correspond to prices—neither locational marginal prices nor DR prices. Therefore, both of these prices must be calculated separately using the results of the optimization. The constraints are still being respected, however, the Lagrangian multipliers cannot simply be taken as prices as the objective function is now the maximization of social welfare. The Lagrangians now represent the change in *social welfare* rather than the change in *cost*, i.e., price.

C. Adaptation of Existing Real-Time Markets

The implementation process described in the preceding section would supplant existing real-time algorithms used by independent system operators (ISOs). In the existing real-time settlement process, as shown in Fig. 10, ISOs collect generator offers and demand forecasts and run their OPF by minimizing total costs subject to constraints. The main output from this process is the generation schedule, which has the selected generator units, time when those units are operating, and quantity of output.

In contrast, the proposed real-time settlement process is depicted in Fig. 11 with new elements in red. ISOs would need to collect DR supply offers in addition to existing generator offers and demand forecasts. The two steps from Fig. 9 now form the settlement engine, with the proposed formulation (14) to (30) in Step 2, where social welfare is maximized.

The litmus test for success is to check if the social welfare increased after Step 2.

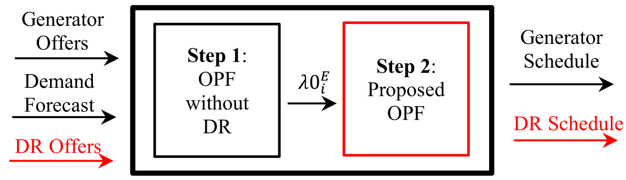


Fig. 11. Proposed real-time settlement process.

TABLE II
SYSTEM PARAMETERS FOR SIMPLE TEST CASE

a	\$0	$p1$	\$5.850e-6/MWh³	PD_T	22,371 MW
b	\$10/MWh	$p2$	-\$0.2939/MWh²	λD^E	\$850/MWh
c	-\$3.502e-7/MWh²	$p3$	\$3,642/MWh		
d	\$2.334e-7/MWh³	$\overline{PR_T}$	8,600 MW		

VI. Results

A. Simple Test Case

We created a simple case study to demonstrate the proposed method to maximize combined social welfare from the energy and DR markets. For this example, a standard quadratic energy supply offer curve $\lambda_S^E(PG_T)$ of the form (32) is used [34]. Note that users can choose a supply offer curve of any form that best suits their system, and this equation should be fitted to the region in which DR is offered, i.e., $PD_T - \overline{PR_T} < PG_T < PD_T$

$$\lambda_S^E\ (PG_T) = \ b + 2 \cdot c \cdot PG_T + 3 \cdot d \cdot {PG_T}^2. \tag{32}$$

By integrating (32), we obtain the total cost for generation, which is needed for (31) in Step 1

$$F_S^E\ (PG_T) = \int_{.}^{PG_T} \lambda_S^E\ (PG)\ dPG$$
$$= a + b \cdot PG_T + c \cdot {PG_T}^2 + d \cdot {PG_T}^3. \tag{33}$$

The DR supply curve $\lambda_S^{\text{DR}}(PR_T)$ used is also quadratic of the form (34). Again, users are free to choose any form of $\lambda_S^{\text{DR}}(PR_T)$ that best suits their system

$$\lambda_S^{\text{DR}}\ (PR_T) = \ p3 + p2 \cdot PR_T + p1 \cdot {PR_T}^2. \tag{34}$$

System parameters are in Table II.

Four scenarios are compared as follows.

S1. OPF without DR: This is a traditional OPF that maximizes social welfare (31) subject to constraints (19) to (29). No DR is considered for this scenario.

S2. Sequential Settlement of Markets: This method settles the DR market first, then it uses the optimum from the DR market to settle the energy market [16], [32]. The first step settles the DR market by finding its Nash equilibrium, where DR social welfare is maximized

$$\text{Maximize DRCS} + \text{DRSS}. \tag{35}$$

TABLE III
RESULTS FOR SIMPLE TEST CASE

	S1. No Demand Response	S2. Sequential Market Settlements [16], [32]	S3. Maximum Net Benefits [15], [30], [31]	S4. Maximum Social Welfare (14) to (30)
Generation Quantity PG_T [MW]	22,371	17,189	18,754	21,437
Demand Response Quantity PR_T [MW]	-	5,182	3,617	934
Energy Price $\lambda_S^E(PG_T)$ [$/MWh]	360.45	216.89	256.28	331.79
Demand Response Price $\lambda_S^{DR}(PR_T)$ [$/MWh]	-	317.93	186.95	29.11
Buyers' Benefit due to DR BBR [$]	-	2,467,679	1,953,570	614,352
Buyers' Cost due to DR BCR [$]	-	1,647,639	676,232	27,196
Net Benefits NtB [$]	-	820,040	1,277,338	587,156
Social Welfare from Energy Market [$]	16,178,373	11,605,473	13,537,451	15,680,364
Social Welfare from DR Market [$]	0	1,881,491	1,703,054	604,946
Total Social Welfare [$]	16,178,373	13,486,964	15,240,505	**16,285,309**

In the second step, the DR quantities PR_i from the DR market are then used as the maximum DR limits $\overline{PR_i}$ when settling the energy market using the objective of maximizing social welfare (31) and constraints (19) to (30).

S3. Maximum Net Benefits: This method considers the NBT required by FERC Order 745 [15], ensuring that benefits BBR (2) exceed costs BCR (3) for buyers of DR. While independent system operators implement NBT using monthly threshold approximations due to technical complexities [30], the same concept could be applied to a real-time OPF by adding new constraint (36) to an energy market settlement of maximizing social welfare (31) and constraints (19) to (30) [31]

$$\mathrm{BBR} - \mathrm{BCR} > 0. \tag{36}$$

S4. Maximum social welfare: Our method co-optimizes the DR and energy markets by maximizing the total social welfare produced in both markets simultaneously, as in (14)–(30).

The numerical results are shown in Table III. Clearly, S4 produces the greatest total social welfare of all the scenarios at $16285309 and is therefore the optimal solution. Interestingly, S2 and S3 yield *less* total social welfare than the base case S1 without DR, and it is, therefore, counterproductive to purchase DR at those levels. In terms of total social welfare, our method S4 offers a 0.66% improvement over S1, a 20.7% improvement over S2, and a 6.86% improvement over S3.

The results are depicted in the combined energy and DR market settlement graph in Fig. 12 and in the social welfare chart in Fig. 13. The dashed vertical lines represent the solutions for each of the four scenarios. Again, this shows that the scenario with the maximum total social welfare is S4.

B. Simple Test Case With Congestion

The simple test case abovementioned was modified by introducing a constrained line that limited the quantity of inexpensive generation available, forcing more expensive local generation to be purchased in the absence of DR. Our proposed method demonstrates how DR can be purchased to alleviate line congestion while maximizing social welfare. The same test case in the preceding section was used, however, a 20 000 MW line limit was introduced, as shown in Fig. 14. One side of the line had inexpensive generation, while the other side of the line had the load and expensive generation. In the absence of DR, line

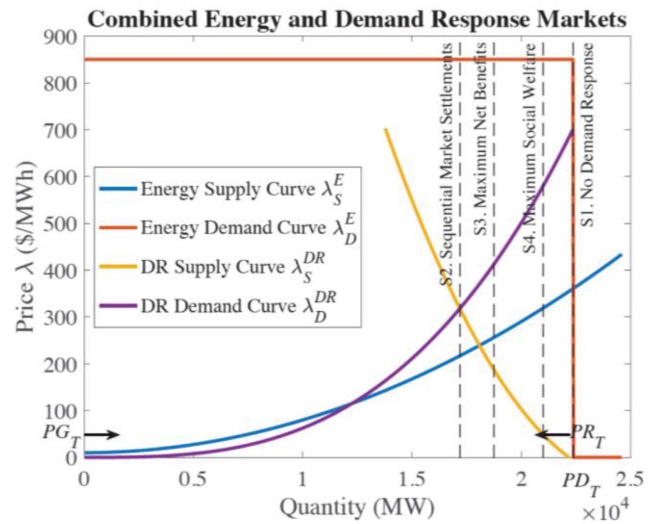


Fig. 12. Combined energy and demand response markets for simple test case.

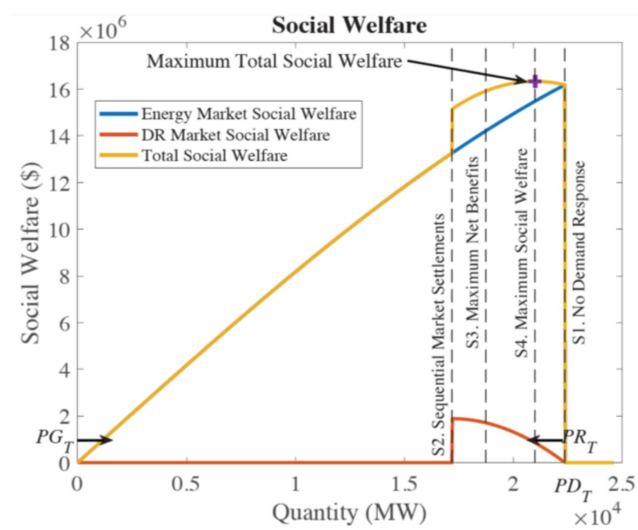


Fig. 13. Social welfare for simple test case.

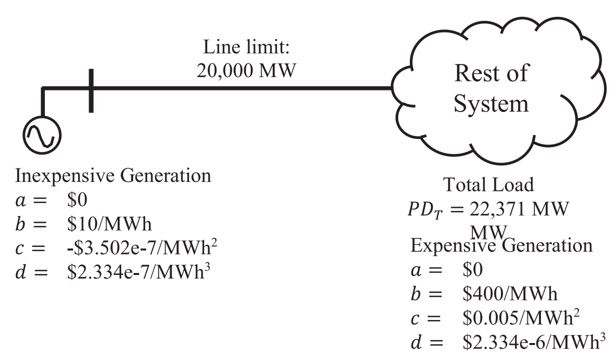


Fig. 14. Simple test case with congestion.

TABLE IV
RESULTS FOR SIMPLE TEST CASE

	S1. No Demand Response	S2. Sequential Market Settlements [16], [32]	S3. Maximum Net Benefits [15], [30], [31]	S4. Maximum Social Welfare (14) to (30)
Total Generation Quantity PG_T [MW]	22,371	16,824	18,099	19,990
Inexpensive Generation Quantity [MW]	19,990	16,824	18,099	19,990
Expensive Generation Quantity [MW]	2,381	0	0	0
Demand Response Quantity PR_T [MW]	-	5,547	4,272	2,381
Social Welfare from Energy Market [$]	15,683,691	13,020,692	13,819,137	14,927,256
Social Welfare from DR Market [$]	-	3,658,687	3,479,405	2,517,548
Total Social Welfare [$]	15,683,691	16,679,379	17,298,541	**17,444,804**

TABLE V
SYSTEM PARAMETERS FOR PJM CASE STUDY

a	$0	$p1$	$1.50E-06/MWh^3	PD_T	140,510 MW
b	-$21,856/MWh	$p2$	-$0.4471/MWh^2	λD^E	$3,000/MWh
c	$0.2832/MWh^2	$p3$	$ 33,243/MWh		
d	-$1.629E-06/MWh^3	$\overline{PR_T}$	28,088 MW		

congestion would force loads to purchase expensive generation locally once the line limit is reached. The aggregate energy supply curve characteristics for both sides are listed in Fig. 14.

Numerical results are in Table IV. Again, the proposed method S4 improves the total social welfare by optimally purchasing DR. This avoids purchasing more expensive local generation due to congestion restrictions. Note also that existing DR procurement methods S2 and S3 recommend purchasing more DR than S4. However, this results in lower total social welfare, so the overpurchasing of DR is counter-productive. In terms of total social welfare, our method S4 offers a 11.2% improvement over S1, a 4.59% improvement over S2, and a 0.846% improvement over S3.

C. PJM Case Study – Peak Demand

To ensure the practicality of our proposed method, we tested our method using PJM's system data [35]. The largest generator units were removed in order to simulate the emergency situation when fuel supply lines are compromised on a day with very high demand. Parameters are in Table V.

Table VI shows the numerical results, again with optimal solution S4 producing the highest total social welfare at $1042.33M. Existing DR procurement methods S2 and S3 produce more social welfare than S1 without DR. S2 and S3 do not perform as well as S4 in terms of social welfare despite procuring more DR. This suggests that existing methods S2 and S3 procure too much DR and should have stopped at S4 levels. In fact, S4 outperforms S1 by 0.47%, S2 by 3.05%, and S3 by 1.41%. For this PJM study, the lost social welfare in that hour for S1, S2, and S3 equal $4.85M, $30.9M, and $14.5M, respectively. Fig. 15 shows the combined energy and DR market settlement graph, while Fig. 16 shows social welfare.

Through our new dual market visualizations, the results of both the simple test case and the PJM case study show that our

TABLE VI
RESULTS FOR PJM CASE STUDY

	S1. No Demand Response	S2. Sequential Market Settlements [16], [32]	S3. Maximum Net Benefits [15], [30], [31]	S4. Maximum Social Welfare (14) to (30)
Generation Quantity PG_T [MW]	140,510	123,613	127,537	135,871
Demand Response Quantity PR_T [MW]	-	16,897	12,973	4,639
Energy Price $\lambda_S^E(PG_T)$ [$/MWh]	607.09	285.38	325.93	473.90
Demand Response Price $\lambda_S^{DR}(PR_T)$ [$/MWh]	-	940.19	666.94	239.99
Buyers' Benefit due to DR BBR [$]	-	39,768,291	35,858,163	18,096,749
Buyers' Cost due to DR BCR [$]	-	15,886,276	8,652,488	1,113,207
Net Benefits NtB [$]	-	23,882,016	27,205,676	16,983,542
Social Welfare from Energy Market [$]	1,037.48M	977.86M	995.67M	1,024.95M
Social Welfare from DR Market [$]	0	33.58M	32.16M	17.39M
Total Social Welfare [$]	1,037.48M	1,011.44M	1,027.82M	**1,042.33M**

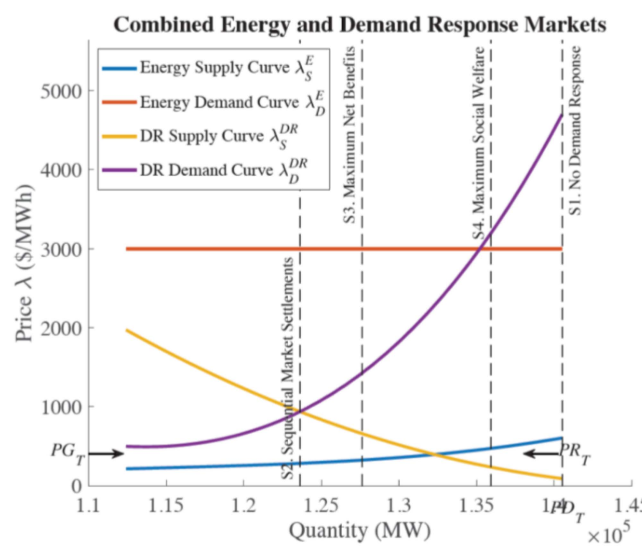


Fig. 15. Combined energy and demand response market settlement graphs for relevant region $0 < PR_T < \overline{PR_T}$ for PJM case study for peak demand.

proposed method S4, which maximizes the total social welfare of the real-time energy and DR markets simultaneously, is the most economically efficient choice for market operations.

D. PJM Case Study – Light Demand

To better illustrate the effects of our proposed approach on a wider range of circumstances, we tested the PJM system under light loading conditions. The parameters were the same as Table V, but with PD_T as 74446.306 MW, which was the minimum load that season, and λD^E as $300/MWh.

In this case, our proposed method recommends that *no DR be procured* because any DR purchase will actually decrease the total social welfare. With any DR purchase, the *gains in social welfare in the DR market* are not enough to offset the *losses in social welfare in the energy market*. The results are depicted in

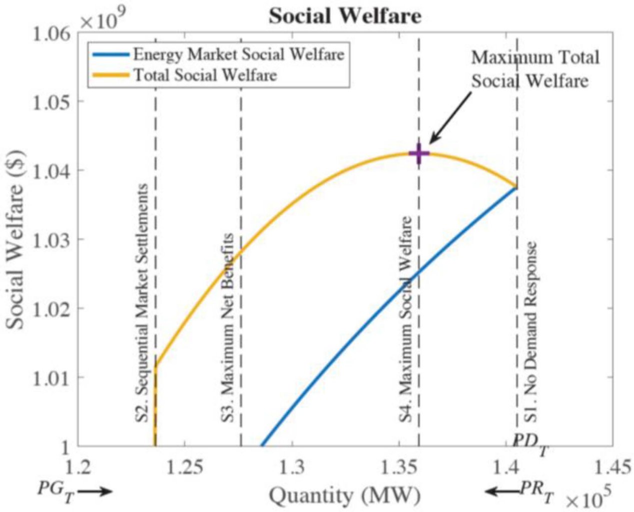


Fig. 16. Social welfare for PJM case study for peak demand.

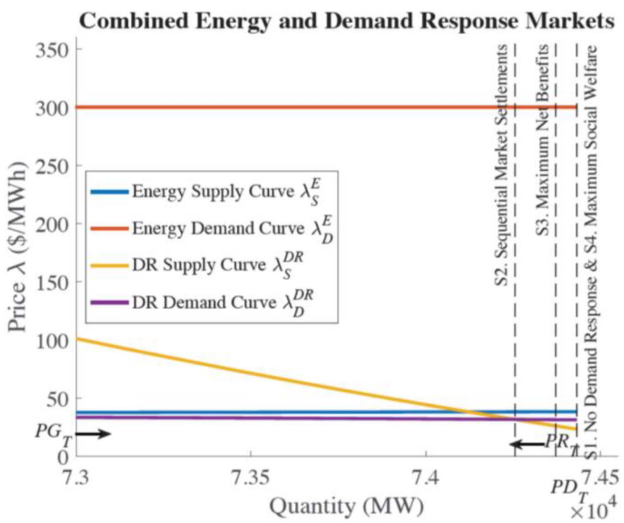


Fig. 17. Combined energy and demand response market settlement graphs for PJM case study for light demand.

Figs. 17 and 18, which are zoomed in for clarity. Fig. 17 provides a combined energy and demand response market settlement graphs for PJM case study for light demand. In Fig. 18, the social welfare gains from DR market are barely visible as they are dwarfed by the energy market's social welfare. Note that existing DR procurement methods S2 and S3 do recommend the purchase of DR, but these would in fact be detrimental to the collective social welfare and economically inefficient. Fig. 18 clearly shows that the maximum social welfare point is when zero DR is procured, and social welfare declines as DR increases, i.e., moving from right to left on the graph.

S4 will always outperform other existing methods in terms of total social welfare as it is the only method that explicitly maximizes for social welfare, considering the two markets. Results for peak and light loading demonstrate the proposed method for extreme cases.

This is an important result as it demonstrates that *DR as a service should only be purchased under one circumstance:* when it can grow the total social welfare. This points to the need to procure DR judiciously, and the case studies show that

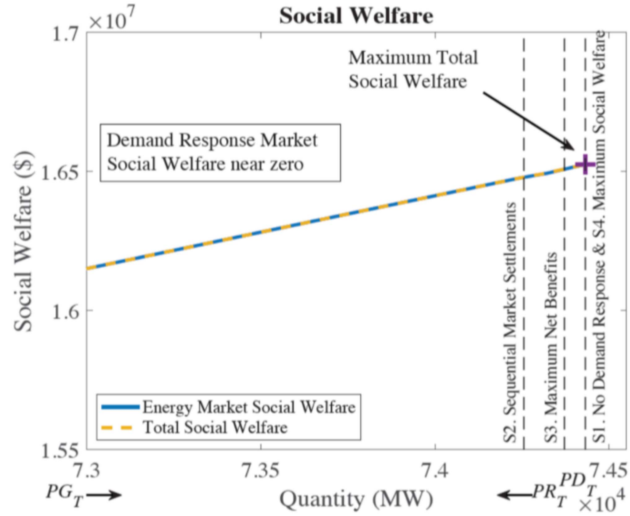


Fig. 18. Social welfare for PJM case study for light demand.

our approach is more restrictive than existing DR procurement methods. However, despite less DR being procured, the total social welfare is more, and therefore, the most economically efficient outcome is achieved through our proposed method.

VII. Conclusion

We present a theoretical model that creates a real-time DR market and co-optimizes energy and DR markets simultaneously. This guides economic DR procurement. DR procurement for other purposes such as reliability is out of scope for this study. DR is a commodity unlike any other procured in the electricity sector. The procurement of DR affects the traditional energy market on which most academic attention is focused, but it also involves the DR market, which is neglected in most discourse about DR.

By maximizing the net benefit to customers of an energy market when procuring DR services, a demand curve for a DR market is generated and thus a DR market is created. The social welfare of the proposed DR market is computed, which is interdependent on the energy market.

Owing to this interdependency, where existence of a DR market is dependent on the energy market, we propose a co-optimization method that simultaneously maximizes the total social welfare of the energy and DR markets. We present a corresponding OPF formulation to implement this method in a real-time scheduling problem. This formulation is applied to four cases: a simple test case to clearly illustrate the mechanics of our proposal; the simple test case with line congestion; and two case studies based on PJM's actual data for both peak and light demand to demonstrate the practicality of our approach. Our method in maximizing total social welfare (S4) was compared to three other existing scenarios found in practice and in the literature: no DR (S1); sequentially settling the DR market and then the energy market (S2); and maximizing net benefits (S3). Both the simple test case and the PJM case study show that S4 is the optimal method, yielding the highest total social welfare, and therefore the most efficient economic outcome. For the PJM case, total surplus for that hour, is improved by \$4.85M, \$30.9M, and \$14.5M when compared with scenarios S1, S2, and S3, respectively.

The results show that DR benefits are limited and can be achieved only up to a certain point. Beyond that point, any additional DR purchase is detrimental. The comparisons with S2 (sequential market settlements) and S3 (maximize net benefits) demonstrate the importance of limiting DR purchases to S4, where total social welfare is maximized.

Future work can entail adapting and applying this novel theory of simultaneously maximizing total social welfare from energy and DR markets to variations in local circumstances. For example, systems with merchant-owned transmission and distribution lines would require merchandizing surplus to be maximized alongside energy and DR market surplus. Applying this work to linear approximations through dc networks could also be a useful application in the future.

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