Mortality Risk Modeling: Applications to Insurance Securitization

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Catastrophic Mortality and Longevity Risks

- Fatalities from natural and man-made catastrophes
  - Flu is 1918 killed at least 40 million people.
  - The earthquake and tsunami in 2004 killed 182,340 people and made 129,897 missing.
  - The 2008 Sichuan earthquake in China killed around 70,000 people.

- Long term effects
  - Public health improvements, such as sanitary sewers and clean water.
  - Tobacco smoking.
  - Physical activity and diet.

- Steady improvement
  - Increased risk for annuity providers; demand for annuities increasing (Mitchell et al., 2001)
  - Defined benefit pension plans.
  - Existing social security system in the US: pay-as-you-go DB plan.
Mortality Securities – Risk Management Tool and Investment Opportunity

- Approaches to manage mortality risks
  - Reinsurance
  - Capital Markets
    - Bigger capacity
    - More flexibility
- A new investment opportunity
  - No or low correlation with traditional capital markets
- Relatively few successes.
A Stochastic Mortality Model Characteristics

- Current general trend
- Deviations from the trend
- Correlations among different ages
- Uneven effects of a mortality jump across different ages
- Both “bad” and “good” shocks
  - Temporary mortality jump
  - Permanent longevity jump
- Flexible, allow for expert judgement.
- Difficulties
  - One population
  - Lack of data
  - Basis risk
A Stochastic Mortality Model Must Reflect

Figure: Actual force of mortality $\mu(x, t)$ for the US males age $x = 35$ (left) and age $x = 75$ (right), for $t = 1901$ to $2005$. 

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Mortality Risk Modeling
Lee and Carter (1992) mortality model

\[ \mu^{LC}(x, t) = \exp[a(x) + b(x)k(t)], \]

where

\[ k(t + 1) = k(t) + g_1 + g_2 \times \text{Flu} + \sigma z(t). \]

Data

- Human Life Table Database and Human Mortality Database
- US male ages \( x = 0, 1, \ldots, 103 \)
- Years \( t = 1901, 1902, \ldots, 2005 \)
Estimated time-series common risk factor $k(t)$

- Data: US male population mortality tables from 1901 to 2005

![Graph showing mortality risk modeling over time]
\[ \mu(x, t) = \mu^{LC}(x, t) \times \exp(-G(x, t) + H(x, t)), \]

- \( \mu^{LC}(x, t) \): the general trend estimated from the Lee and Carter (1992) model.
- Permanent Longevity Jump \( G(x, t) \)
- Temporary Adverse Mortality Jump \( H(x, t) \)
Permanent Longevity Jump $G(x, t)$

\[ G(x, t) = K(x, t) + D(x, t). \]

- Jump reduction component $K(x, t)$: one-time permanent jump

\[ K(x, t) = \sum_{s=1}^{\infty} y_s A_s(x) 1\{t \geq \eta_s\}, \]

- $\eta_s$: time of jump reduction event $s$
- $y_s > 0$: maximum mortality improvement of all ages in jump reduction event $s$
- $A_s(x) \in [0, 1]$ distributes the effects of event $s$ across different ages $x$. 
Permanent Longevity Jump $G(x, t)$

$$G(x, t) = K(x, t) + D(x, t).$$

- Trend reduction component $D(x, t)$: steeper downward-sloping mortality curve

$$D(x, t) = \sum_{i=1}^{\infty} \zeta_i(t - \nu_i)F_i(x)\exp(-\xi_i(t - \nu_i))1_{\{t\geq\nu_i\}},$$

- $\nu_i$: time of trend reduction event $i$
- $\zeta_i > 0$: maximum annual excess mortality improvement of all ages in trend reduction event $i$
- $F_i(x) \in [0, 1]$ distributes age effects.
- $(t - \nu_i)$ accumulates this mortality improvement effect.
- $\xi_i > 0$ specifies the length of trend reduction event $i$. 
Temporary Mortality Jump $H(x, t)$

\[
H(x, t) = \sum_{j=1}^{\infty} b_j B_j(x) \exp(-\kappa_j(t - \tau_j)) \mathbb{1}_{\{t \geq \tau_j\}},
\]

where

- $\tau_j$: time of adverse mortality event $j$
- $b_j > 0$: *maximum* mortality deterioration among all ages in jump event $j$.
- $B_j(x) \in [0, 1]$ distributes mortality jump impact across ages.
- $\exp(-\kappa_j(t - \tau_j))$ where $\kappa_j > 0$ models transitory nature of mortality jumps.
Parsimonious Model

\[ \mu(x, t) = \exp[a(x) + b(t)k(t) - G(x, t) + H(x, t)] \]
\[ = \exp[a(x) + b(x)k(t)] \]
\[ \times \exp \left\{ -\zeta F(x) \sum_{i=1}^{M(t)} (t - \nu_i) \exp(-\xi(t - \nu_i)) \mathbf{1}_{\{t \geq \nu_i\}} \right\} \]
\[ \times \exp \left\{ bB(x) \sum_{i=1}^{N(t)} \exp(-\kappa(t - \tau_j)) \mathbf{1}_{\{t \geq \tau_j\}} \right\}, \]

where

- we exclude the longevity jump reduction component \( K(x, t); \)
- we assume constant mortality and longevity jump effects for age \( x. \)
Parameter Estimates for Different Ages

Left: $F(x)$ – the normalized annual excess percentage decrease in $\mu(x, t)$ of different ages in the 1970’s.
Right: $B(x)$ – the impact of the 1918 worldwide flu on $\mu(x, t)$ across ages. The $x$-axis of all three graphs represents the age.
Simulated Sample Paths of Force of Mortality: Ages 30, 40, 50 and 60
Proposed Longevity Risk Hedging

- J. P. Morgan $q$-forward
- At maturity $t + 10$, the payoff of a longevity call option on $10p_{65,2005}$ (10-year survival rate of age 65 at time $t = 2005$) with notional amount $100,000,000$:

$$C = \begin{cases} 100,000,000(10p_{65,2005} - p) & \text{if } 10p_{65,2005} > p \\ 0 & \text{otherwise} \end{cases}$$

- At maturity $t + 10$, the payoff of a longevity tranche on $10p_{65,2005}$ (10-year survival rate of age 65 at time $t = 2005$) with notional amount $100,000,000$:

$$B = \begin{cases} 100,000,000(p_2 - p_1) & \text{if } 10p_{65,2005} \geq p_2 \\ 100,000,000(10p_{65,2005} - p_1) & \text{if } p_1 < 10p_{65,2005} < p_2 \\ 0 & \text{if } 10p_{65,2005} \leq p_1 \end{cases}$$
Present value of future annuity benefits for an annuity written on $M$ lives ($x_i$) at time 0, for $i = 1, 2, \ldots, M$ is

$$X_i = \bar{s} \sum_{k} e^{-kr_k} \mathbb{I}\{T(x_i) \geq k\},$$

Aggregate present value is

$$Y = \sum_{i=1}^{M} X_i.$$

The premium $P$ per policy is to be determined from the equation:

$$E[u(w + MP - Y)] = u(w).$$
We use the exponential utility function

\[ u(w) = \frac{1 - e^{-\alpha w}}{\alpha}, \]

where \( \alpha \) is the risk aversion parameter of the insurer.

An approximation for the premium \( P \) per policy, which can be found in Gerber and Pafumi (1998), can be made as

\[ P \approx \frac{1}{M \alpha} \left[ \alpha E[Y] + \frac{1}{2} \alpha^2 \text{Var}(Y) \right], \]

where

\[ \text{Var}(Y) = E[\text{Var}(Y|\Theta)] + \text{Var}[E(Y|\Theta)] \]
\[ = E[M \text{Var}(X|\Theta)] + \text{Var}[ME(X|\Theta)] \]
\[ = ME[\text{Var}(X|\Theta)] + M^2 \text{Var}[E(X|\Theta)]. \]
Estimating Risk Aversion Parameter $\alpha$

- The Genworth Financial Group sold around $560 million of single premium individual annuities in the US in 2005 with a monthly payout rate of $6.40 per $1,000 premium (Stern 2008).
- We assume that the policies are identical, issued to males age 65 with a gross premium of $250,000.
- Then the number of policies is $M = 560 \text{ million}/250,000 = 2,240$. 

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Mortality Risk Modeling
We assume the underlying expense factor is 12.25 percent, which is around the industry average, so the net premium per policy to the insurer is

\[ P = 250,000 \times (1 - 0.1225) \]
\[ = $219,375. \]

We estimate the risk aversion parameter \( \alpha \) with a “Monte Carlo simulation within a Monte Carlo simulation.”

We obtain \( \alpha = 4.8 \times 10^{-7} \).
Exponential Utility Model with $\alpha = 4.8 \times 10^{-7}$

Figure: Genworth’s estimated utility function under the exponential utility model for $\alpha = 4.8 \times 10^{-7}$. The vertical axis represents utility and the horizontal axis represents wealth, both in thousands.
Table: 10-year longevity call option prices with data generating process driven by our model.

<table>
<thead>
<tr>
<th>Strike rate $p$</th>
<th>0.775</th>
<th>0.780</th>
<th>0.785</th>
<th>0.790</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call price</td>
<td>$1,121,364</td>
<td>$924,288</td>
<td>$772,136</td>
<td>$652,506</td>
</tr>
</tbody>
</table>

Table: 10-year longevity call option prices with data generating process driven by the Lee-Carter model alone.

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<tr>
<th>Strike rate $p$</th>
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<th>0.790</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call price</td>
<td>$191,962</td>
<td>$77,070</td>
<td>$23,864</td>
<td>$5,522</td>
</tr>
</tbody>
</table>
Range of Outcomes for $10p_{65,2005}$ with Our Model

Figure 11. Range of outcomes for $10p_{65,2005}$, 2005 under the full model.

Figure 12 shows a histogram of simulated values of $10p_{65,2005}$ for 1 million draws using the Lee-Carter model alone. Evidently, it is the right tail behavior that is driving the differences between the prices of the longevity calls across the two mortality data generating processes.
Range of Outcomes for $10p_{65,2005}$ with Lee-Carter Model

FIGURE 12. Range of outcomes for $10p_{65,2005}$, 2005 under the Lee-Carter model alone.

The dollar payoff from this longevity call spread at maturity in 2015 may be expressed as

$$B = \begin{cases} 
100,000,000 (p_2 - p_1) & \text{if } 10p_{65,2005} \geq p_2 \\
100,000,000 (10p_{65,2005} - p_1) & \text{if } p_1 < 10p_{65,2005} < p_2 \\
0 & \text{if } 10p_{65,2005} \leq p_1 
\end{cases}$$

Table 5 shows the longevity call spread option prices for a range of strikes for mortality driven by our model. Note that the spreads in Table 5 have different, decreasing seniorities due to decreasing trigger and exhaustion levels. In particular, spread A has the lower attachment and detachment points relative to the other spread options and thus it demands a higher price.

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Mortality Risk Modeling
Related issues

- General insurance asset-liability risk management
  - Non-life business returns and asset returns considered together
  - Markowitz method and analysis of tails
  - Forthcoming *NAAJ*

- Life insurance and annuity asset-liability risk management
  - One population is inadequate. (Cairns, Longevity 5).
    - Population 1 – Index for security, 2 – Pension plan
    - Life company’s four populations: Male annuitants, male lives insured, female annuitants, female lives insured
  - One period model may not be adequate.
Conclusions

- We have described the mortality evolution as a dynamic process:
  - a general mortality trend
  - a diffusion process
  - correlation among different ages
  - uneven effects of a mortality jump across different ages
  - a “temporary” mortality jump process and a “permanent” longevity jump process

- We illustrated a parsimonious version of our model.

- We applied our model to securitization.

- There is a lot more work to do.