An Investment Model
via Regime Switching Economic Indicators

Yonggan Zhao
Canada Research Chair (Tier II) in Risk Management
School of Business Administration
Dalhousie University
Halifax, Nova Scotia, Canada

Coauthored with Professor John M. Mulvey
1 Introduction

1.1 Objectives

• To develop a dynamic asset pricing model with regime-switching economic indicators, which characterize market sentiments.

• To develop an investment model with exchange traded funds, incorporating control for losses and risk exposure.
1.2 The equity market

Equity market performance appears to be persistent in a medium time range within a market sentiment.

Figure 1: S&P 500 Index Returns Over Time
The implied volatility curve by the S&P 500 index options is highly skewed, rejecting the random walk assumption for stock prices.
1.3 Financial instruments

In 1998, State Street Global Advisors introduced the “Sector Spiders”, which follow the nine sectors of the S&P 500.

Figure 2: Select Sector ETFs

ETFs generally provide an easy diversification, low expense ratios, and tax efficiency, while still maintaining all the features of ordinary stock, such as limit orders, short selling, and options.
1.4 **Economic Indicators**

- **Current stock market return (STK):** Log return on the S&P 500 Price Index.
- **Current bond market return (BND):** Log return on the 10 Year U.S. Treasury Bond.
- **Current currency strength (USD):** Log changes in the Dollar Index.
- **Volatility (VIX):** Measured as the standard deviation of short term stock returns.
- **Dividend yield (EDY):** S&P 500 Aggregate Dividend Yield.
- **Interest rate (UIR):** U.S. Interbank Offer Rate.
- **Yield spread (TYS):** 10 year U.S. Treasury Bond - 3 Month T-Bill.
- **Credit spread (UCS):** U.S. Corporate BAA - U.S. Corporate AAA.
Stocks and VIX move in opposite directions (correlation: -75%). Stocks and bonds sometimes move together. Most of the time, stocks and the U.S. currency move in opposite directions.
Interest rate and yield spread move in opposite directions. Credit spread is usually high, when interest rate is low.
2 A Regime Switching Model for Economic Indicators

2.1 Non-normality returns

Figure 3: Empirical distribution of the monthly returns on the S&P 500 index from January 1990 to September 2009.
Implications:

- Returns do not appear to be normal and exhibit skewness and kurtosis.
- There appear two modes for the observed monthly returns – one negative and one positive – indicating the difference between a bull and a bear market.
- Returns around zero are in a lower frequency than returns in both positive and negative ends, which reinforces the idea of a mixture normal distribution.
- Returns in the left tail are much fatter than those in the right tail.
2.2 Probability distribution with regime-switching models

Suppose factor changes are mixture of normals generated by a Markov regime switching process and there exist $K$ regimes representing possible economy sentiments over time.

- For a given regime, factor changes follow a multivariate normal distribution, $N(\mu_k, \Sigma_k)$, $k = 1, 2..., K$.

- The transition probability from regime $i$ to $j$ is constant and denoted by
  \[ P = (p_{ij} = \Pr(j|i)) \]

- The return in the next period is distributed as a mixture of normals. Given regime $i$ at time $t - 1$, the conditional density function of return at time $t$ is
  \[ p_{i1}N(\mu_1, \Sigma_1) + p_{i2}N(\mu_2, \Sigma_2) + \cdots + p_{iK}N(\mu_K, \Sigma_K). \]

- Since regimes are not observable, the regime distribution must be dynamically updated.
• How close is a mixture model to reality? Below is a plot of a 3-regime density function estimated from the real life data presented in Figure 4.

With mixing parameters \((0.3, 0.4, 0.3)\) and the component distribution

\[ N(-0.3, 0.3), N(0, 0.15), N(0.3, 0.1) \]
2.3 **Optimal number of regimes**

The Bayesian Information Criterion (BIC) is an asymptotic result derived under the assumptions that the data distribution is in the exponential family.

Let:

- $R$ = the observed data;
- $N$ = the number of data points in $R$;
- $Z(K)$ = the number of model parameters to be estimated for $K$ regimes.
- $L(K)$ = the maximized value of the likelihood function.

The formula for the calculation of the BIC index is:

$$BIC(K) = -2 \cdot \ln L(K) + Z(K) \ln N.$$  

To find the optimal number of regimes, we need to minimize $BIC(K)$ over a set of the selected $K$. 

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2.4 **Autoregressive regime-switching risk factors**

Let $X_t$ be the vector of the selected risk factors at time $t$. We assume that $F_t$ follows a regime-switching VAR(1) model:

$$X_t = \alpha_{M_t} + X_{t-1} \cdot \beta_{M_t} + \gamma_{M_t} \epsilon_t$$

where $M_t$ is a Markov chain with $K$ regimes representing the situation of the economy over time, such as “Bull Market”, “Bear Market”, or “Flat Market”, etc.

$\alpha_{M_t}$, $\beta_{M_t}$ and $\gamma_{M_t}$ are regime-dependent coefficients.

$\epsilon_t$ is an iid process with a standard multivariate normal distribution.

Uncertainty arises from both $\epsilon_t$ and $M_t$. 
2.5 A regime-switching regression model for asset returns

Having presented a regime-switching model for risk factors, we now discuss how investment asset returns are related to the risk factors.

We again assume that returns of all primary investment assets follow a linear model with regime-dependent risk sensitivity.

Our interest is to develop a predictive model for asset returns based on the forecast of the risk factors.

Explicitly, the following structure is specified:

\[ R_t = A_{M_t} + \hat{X}_{t,M_t} \ast B_{M_t} + \Gamma_{M_t} \theta_t \]

where \( \hat{X}_{t,M_t} \) is the regime-dependent forecast value of the risk factors. \( A_{M_t}, B_{M_t} \) and \( \Gamma_{M_t} \) are also regime-dependent coefficients. \( \theta_t \) is an iid process with a standard multivariate normal distribution.
The dynamics of asset returns are linearly related to the prediction of risk factors.

As regimes are not observable, the unconditional joint probability distribution of the returns is a multivariate mixture of normals with mixing parameters equal to the prior distribution of the regime at time $t$.

Denote the posterior probability of regime $i$ at time $t - 1$ by $q_{t-1}(i)$, then the prior probability of the regime being $k$ at $t$ is

$$p_t(k) = p_{1k}q_{t-1}(1) + p_{2k}q_{t-1}(2) + \cdots + p_{Kk}q_{t-1}(K)$$

Hence, the unconditional expected return from $t - 1$ to $t$ is

$$E[R_t] = E[E[R_t|M_t]] = \sum_{k=1}^{K} (A_k + \bar{X}_{t,k}B_k)p_t(k)$$

with a covariance matrix

$$V[R_t] = \sum_{k=1}^{K} p_t(k)\Gamma_k + \sum_{k=1}^{K} p_t(k) (E[R_t|M_t = k] - E[R_t])^2.$$
3 Optimal Investment Strategies

Let $W_t$ be the portfolio weight vector (fraction of investment capital) in the $Q$ financial instruments from time $t - 1$ to $t$. Since each of the individual financial instruments follows a mixture of normal distribution, the portfolio return is also mixture of normally distributed with mixing parameters being the prior probability distribution of the regimes at time $t$. Thus, the expected return of a portfolio, denoted as $E_t(W)$ in period $t$ is

$$E_t(W) = r_{f_t} + (E[R_t] - r_{f_t})^\top W_t$$

where $r_{f_t}$ is the risk free rate. The variance of the portfolio, denoted as $V_t(W)$, is

$$V_t(W) = W_t^\top V[R_t] W_t$$

With an investment horizon, say $T$ periods, decisions on portfolio weights are made in each time period. Denoting the portfolio return with weight $W_t$ at time $t$ by $R_t(W)$, we now define the portfolio model.
**The Objective Function:** Suppose the investor intends to maximize the expected return of the portfolio with a penalty on the shortfalls. Thus, the objective function is then written as

$$\max_W E_t(W) - \lambda(E_t[b_t - R_t(W)]^+)$$

where $b_t$ is the target return and $\lambda$ is a constant for the size of penalty.

**A Probabilistic Constraint:** Financial institutions are required to put aside a portion of their risk portfolio as capital reserve. This is quantified using the Value-at-Risk measure (e.g., Rockafellar and Uryasev 2000). Portfolio return is constrained not to be lower more than some percentage from its target return with some maximum probability. The mathematical expression is

$$\Pr\{b_t - \tilde{R}_t(W) \geq \rho_t\} \leq \eta_t$$

where $\rho_t$ is the percentage of shortfall from portfolio’s target return $b_t$ and $\eta_t$ is the maximum probability of the shortfall.

**Constraints for Risk Exposures:** Portfolio weights are very sensitive to the changes in risk factors (Best and Grauer, 1991; Chopra and Ziemba, 1993). The
investment portfolio is constrained so that its risk exposure, measured as the coefficient to each of the risk factors, is restricted to be within a suitable range. Thus, at any point in time, the portfolio return is expressed as

\[ R_t(W) = (A_{M_t} + X_{t,M_t}B_{M_t}) * W_t + \phi_{M_t} * W_t \]

The regime-dependent portfolio risk exposures to all the indicators, defined as \( B_{M_t}W_t \), are constrained as

\[ \left| \sum_{k=1}^{K} p_t(k) * B_k * W_t \right| < \delta \]  

(1)

where \( \delta \) is a positive constant to gauge the maximum risk exposure.

**Bounds for Portfolio Weights**: In order to control portfolio’s turnover, lower and upper bounds for investments in individual financial instruments are imposed. In addition, limits of total long and short positions are also included as constraints in the portfolio selection process. The limits on the portfolio weights stabilize the investment strategy.
Mathematical representation of the optimization model:

$$\max_{W} \quad E_t(W) + \lambda \sum_{k=1}^{K} p_t(k) \int_{-\infty}^{z_k(b_t, \rho_t, W)} \left( E_{tk}(W) + z \frac{1}{V_{tk}^2(W)} - b_t \right) f(z) \, dz$$

subject to

$$\left| W^\top B_n \right| \leq \delta_n, \quad \forall n = 1, \ldots, N.$$  
$$\sum_{k=1}^{K} \Pr(M_t = k) \int_{-\infty}^{z_k(b_t, \rho_t, W)} f(z) \, dz \leq \phi_t$$

$$l \leq W \leq u$$

(2)

where

$$z_k(b_t, \rho_t, W) = \frac{b_t - \rho_t - E_{tk}}{V_{tk}}$$

$$E_{tk}(W) = E_t[\tilde{R}_t(W) | M_t = k, X_1, \ldots, X_{t-1}]$$

$$V_{tk}(W) = V_t[\tilde{R}_t(W) | M_t = k, X_1, \ldots, X_{t-1}]$$

and \(f(z)\) is the standard normal density function.
4 Parameter Estimation Algorithm

Let $\Theta$ be the set of parameters $\{\alpha_{M_t}, \beta_{M_t}, \Sigma_{M_t}, P_{ij}\}$.

For exposition, we drop the time subscripts. Denote $M$ the space of all possible regime sequences for the time period. The maximum marginal log-likelihood is expressed as

$$\max_{\Theta} \left\{ \ln \sum_{M \in M} P(X, M; \Theta) \right\}, \quad (3)$$

where $P(X, M; \Theta)$ is the joint probability distribution function of $X$ and $M$. From Jensen’s inequality,

$$\ln \sum_{M \in M} P(X, M; \Theta) \geq \sum_{M \in M} Q(M) \ln \frac{P(X, M; \Theta)}{Q(M)}, \quad (4)$$

where $Q$ is an arbitrary distribution on $M$.

We try to find a tight bound by looking into the right hand side of the inequality.
The E-Step. A standard optimization technique implies the optimal solution to

\[
\max_Q \left\{ \sum_{M \in \mathcal{M}} Q(M) \ln \frac{P(X, M; \Theta^0)}{Q(M)} \right\}
\]

is

\[ Q^*(M) = P(M|X; \Theta^0) \]

for the observed data \( X \) and the current estimate \( \Theta^0 \).

Substituting the “optimal” \( Q^*(M) \) in (4) shows that the lower bound function achieves the log-likelihood \( \ln P(X; \Theta^0) \) of the observed data for the current parameter estimate \( \Theta^0 \).

If \( \Theta \) is the true maximum likelihood estimate, then

\[ \ln P(X, M; \Theta) \geq \ln P(X, M; \Theta^0) \]

for any approximation \( \Theta^0 \) and

\[ \ln P(X; \Theta) \geq \sum_{M \in \mathcal{M}} P(M|X; \Theta^0) \ln \left( \frac{P(X,M;\Theta)}{P(M|X;\Theta^0)} \right) \geq \ln P(X; \Theta^0). \]
The M-step. We want to improve the current estimate $\Theta^0$ by maximizing the middle term in the above inequality. This is equivalent to maximizing the expected log-likelihood of the joint data of $X$ and $M$ with respect to $\Theta$

$$E^{Q^*}[\ln P(X, M; \Theta)] = \sum_{M \in M} P(M|X; \Theta^0) \ln P(X, M; \Theta).$$

Hence, an improved estimate for the parameter $\Theta$ is

$$\Theta^1 = \arg \max_{\Theta} \{ E^{Q^*}[\ln P(X, M; \Theta)] \} . \tag{5}$$

The algorithm is guaranteed to increase the log-likelihood at each iteration.

1. E-Step: Set an initial value $\Theta^0$ for the true parameter set $\Theta$, calculate the conditional distribution function, $Q^*(M) = P(M|X; \Theta^0)$, and determine the expected log-likelihood, $E^{Q^*}[\ln P(X, M; \Theta)]$.

2. M-Step: Maximize the expected log-likelihood with respect to the conditional distribution $Q^*$ of the hidden variable to obtain an improved estimate of $\Theta$. The improved estimate is

$$\Theta^1 = \arg \max_{\Theta} \{ E^{Q^*}[\ln P(X, M; \Theta)] \} . \tag{6}$$
5 Empirical Analysis

5.1 Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>STK</th>
<th>BND</th>
<th>USD</th>
<th>VIX</th>
<th>EDY</th>
<th>UIR</th>
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<th>UCS</th>
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<td>0.0010</td>
<td>0.0010</td>
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<table>
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<td>-0.2995</td>
<td>-0.7372</td>
<td>-0.2484</td>
<td>0.1610</td>
<td>0.0870</td>
<td>-0.1268</td>
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<tr>
<td>BND</td>
<td>-0.1722</td>
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<td>-0.1264</td>
<td>0.1740</td>
<td>0.0479</td>
<td>-0.2705</td>
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<td>VIX</td>
<td>-0.7372</td>
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<td>0.1923</td>
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<tr>
<td>TYS</td>
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<td>-0.0641</td>
<td>0.0451</td>
<td>-0.5942</td>
<td>1.0000</td>
<td>0.5550</td>
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<tr>
<td>UCS</td>
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<td>0.1195</td>
<td>0.1813</td>
<td>0.1292</td>
<td>0.0856</td>
<td>-0.7715</td>
<td>0.5550</td>
<td>1.0000</td>
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The assumption of normality for the indicators are strongly rejected!
5.2 The optimal number of regimes

<table>
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<tr>
<th>No. of Regimes</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>-1.2984</td>
<td>-1.2598</td>
<td>-1.2318</td>
<td>-1.2255</td>
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<td>BIC</td>
<td>28903</td>
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<td>26623</td>
<td>26564</td>
<td>26952</td>
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</table>

Thus, the optimal number of regimes is 4.
5.3 Regime inference

- Transition probability matrix

\[
P = \begin{bmatrix}
0.7613 & 0.2254 & 0.0000 & 0.0133 \\
0.0629 & 0.8562 & 0.0630 & 0.0179 \\
0.0000 & 0.0625 & 0.9191 & 0.0183 \\
0.0424 & 0.0000 & 0.0435 & 0.9141
\end{bmatrix}
\]

In long run:
- Regime 1: 12.16%, Regime 2: 34.80%, Regime 3: 36.17%, Regime 4: 16.87%
- **Factor performance by regime**: Average value weighted by the posterior probabilities.

<table>
<thead>
<tr>
<th>Regime</th>
<th>STK</th>
<th>BND</th>
<th>USD</th>
<th>VIX</th>
<th>EDY</th>
<th>UIR</th>
<th>TYS</th>
<th>UCS</th>
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<tr>
<td>1</td>
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<td>0.2764</td>
<td>-0.0478</td>
<td>2.5495</td>
<td>0.9347</td>
<td>-10.6100</td>
<td>5.7489</td>
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<td>2</td>
<td>-0.0018</td>
<td>0.0628</td>
<td>0.0750</td>
<td>-0.9062</td>
<td>0.0994</td>
<td>0.4585</td>
<td>0.7590</td>
<td>-1.0316</td>
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<tr>
<td>3</td>
<td>0.1401</td>
<td>0.1239</td>
<td>-0.1521</td>
<td>0.0347</td>
<td>0.0011</td>
<td>1.3471</td>
<td>-1.9222</td>
<td>0.2192</td>
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<tr>
<td>4</td>
<td>0.3518</td>
<td>0.0253</td>
<td>-0.2053</td>
<td>-0.1720</td>
<td>-0.3125</td>
<td>-0.3074</td>
<td>-0.2540</td>
<td>-0.1745</td>
</tr>
</tbody>
</table>

**Regime 1**: Bear,

**Regime 2**: Transit A

**Regime 3**: Transition B

**Regime 4**: Bull.
5.4 Model Predictability

Figure 4: Predicted and Observed S&P 500 Index
Figure 5: Predicted and Observed XLY Fund Returns

Root Mean Square Errors in Prediction for the ETFs:

<table>
<thead>
<tr>
<th>Model</th>
<th>XLY</th>
<th>XLP</th>
<th>XLE</th>
<th>XLF</th>
<th>XLV</th>
<th>MOO</th>
<th>XLB</th>
<th>XLK</th>
<th>XLU</th>
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<td>Regime Switching</td>
<td>0.0319</td>
<td>0.0213</td>
<td>0.0368</td>
<td>0.0407</td>
<td>0.0244</td>
<td>0.0340</td>
<td>0.0359</td>
<td>0.0373</td>
<td>0.0254</td>
</tr>
<tr>
<td>Single Regime</td>
<td>0.0341</td>
<td>0.0220</td>
<td>0.0385</td>
<td>0.0440</td>
<td>0.0257</td>
<td>0.0372</td>
<td>0.0382</td>
<td>0.0400</td>
<td>0.0264</td>
</tr>
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</table>
Inferred market regimes over time

Figure 6: S&P 500 Index with Regimes

Red: Bear Market
Blue: Transition B
Magenta: Transition A
Green: Bull Market
5.5 The investment model

- starting wealth: \( W = $1000 \)
- Penalty size: \( \lambda = 20 \)
- Maximum probability of drawdown: \( \rho = 0.03 \)
- Wealth security: \( b_t = \text{target 95\% of the current wealth level} \)
- Portfolio risk exposure: \( \delta = 0.5 \)
- Short sales: \( l = -10\% \)
- Investment percentage in each of the ETFs: \( u = 30\% \)
- Portfolio rebalance frequency: weekly
- Leverage: 40\%
- Maximum limit: 50\%.
Portfolio Performance

Figure 7: Portfolio Performance ($\delta = 0.5$)

- SP500+Cash: Investments in the S&P 500 Index and cash.
- Equally weighting portfolio weights are the same as the S&P 500 Index component weights.
## The Sharpe Ratios

<table>
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<th>Model</th>
<th>Annual Growth</th>
<th>Risk</th>
<th>Sharpe Ratio</th>
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<td>Regime Switching</td>
<td>38.84%</td>
<td>17.41%</td>
<td>2.0764</td>
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<tr>
<td>Single Regime</td>
<td>15.65%</td>
<td>15.60%</td>
<td>0.8333</td>
</tr>
<tr>
<td>SP500 + Cash</td>
<td>19.61%</td>
<td>14.88%</td>
<td>1.1002</td>
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<tr>
<td>Equally Weighting</td>
<td>0.24%</td>
<td>18.81%</td>
<td>-0.1281</td>
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## Annual Growth Rate by Varying $\delta$

<table>
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<th>$\delta = 0.1$</th>
<th>$\delta = 0.5$</th>
<th>$\delta = 1.0$</th>
<th>$\delta = 1.5$</th>
<th>$\delta = 2.0$</th>
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<tr>
<td>Regime Switching</td>
<td>7.71%</td>
<td>21.36%</td>
<td>38.84%</td>
<td>40.55%</td>
<td>41.42%</td>
<td>41.25%</td>
</tr>
<tr>
<td>Single Regime</td>
<td>9.96%</td>
<td>14.76%</td>
<td>15.65%</td>
<td>17.57%</td>
<td>18.93%</td>
<td>20.69%</td>
</tr>
</tbody>
</table>
6 Conclusion

- A regime-switching factor model (AR(1)) was appropriate for characterizing market regimes.
- A regime-switching regression model was discussed which links factor indicators to asset returns.
- Market data (Exchange Traded Funds) were used to test the model and the implication of the model to real world investment was strong.
- Further research interests would be in the design of a balanced portfolio of mixed investment asset classes.