Bipartiteness Testing

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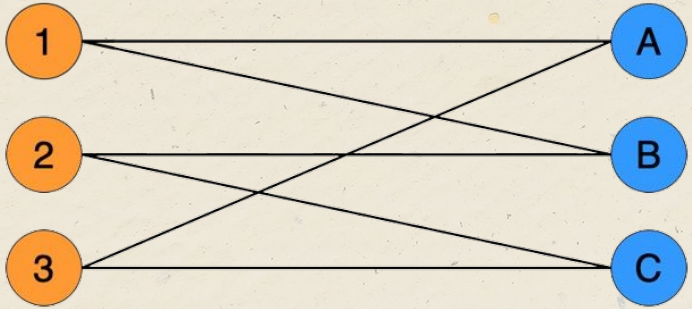
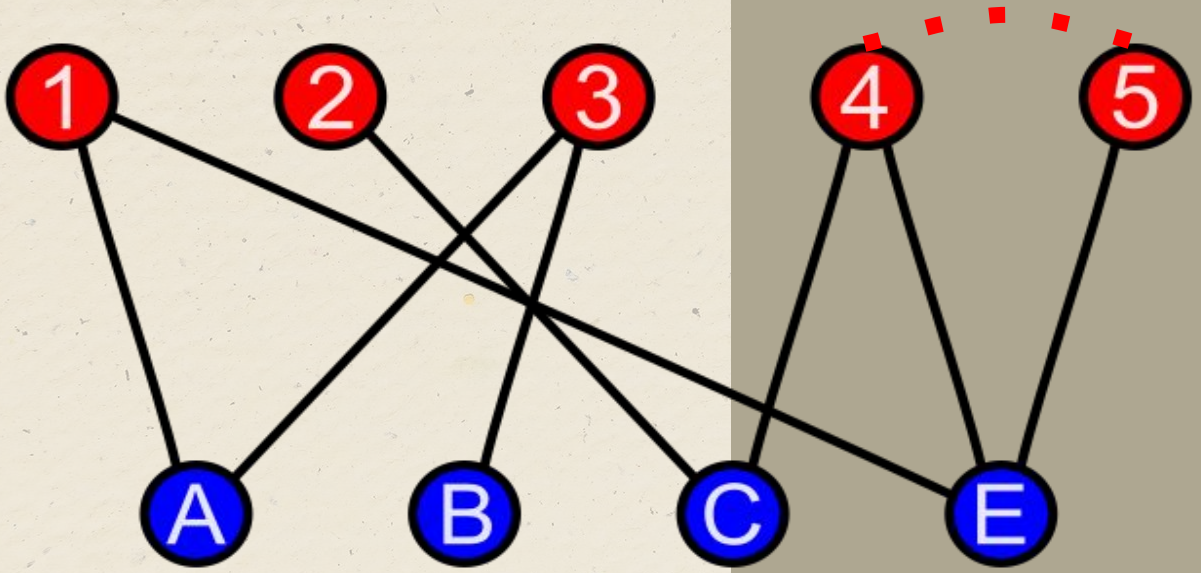
What is Bipartiteness?



“Distance” to Bipartite



A graph G on n vertices is ϵ-**far** from a graph property P if we need to add or remove at least ϵn^2 edges from G to obtain a graph that does have the property P

Standard Decision Maker vs. Property Tester: Accuracy

Decision Making Algorithm	Accepts 100%	Rejects 100%	Rejects 100%
	G has property P	**G is ϵ-close to P**	**G is ϵ-far from P**
Property Testing Algorithm	Accepts ⅔		Rejects ⅔

Complexity

Time complexity vs. Query complexity

Input Size >> Query Complexity

The query complexity of bipartiteness testing does not depend on the size of the input [Goldreich, Goldwasser, Ron '98]

Known to Date

$$O\left(\frac{1}{\epsilon^2}\right) \Longrightarrow o\left(\frac{1}{\epsilon^2}\right)?$$

If a graph is ϵ-far from bipartite, then a random sample on $O\left(\frac{1}{\epsilon}\right)$ vertices is not bipartite with high probability

[Alon, Krivelevich '02] proved this and used it to prove the current best tester

If a graph is ϵ-far from bipartite, then a random sample on $O\left(\frac{1}{\epsilon}\right)$ vertices is $\Omega(\epsilon)$-far from bipartite with high probability

Conjectured by [Bogdanov, Li, '10] and can be used to show a better tester if proven

The Bogdanov/Li conjecture holds true when the graph is regular

Bogdanov and Li, 2010

And in the case when all the vertices in the graph have low-degree or all the vertices have high-degree

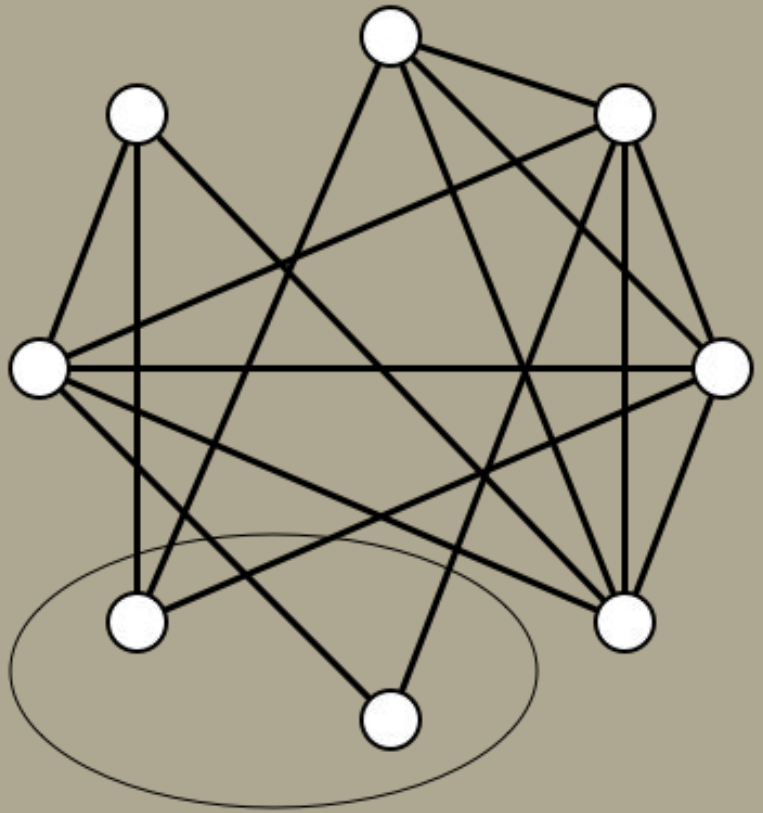
Gonen and Ron, 2010

The Container Method to Improve Bounds on Bipartiteness Testing

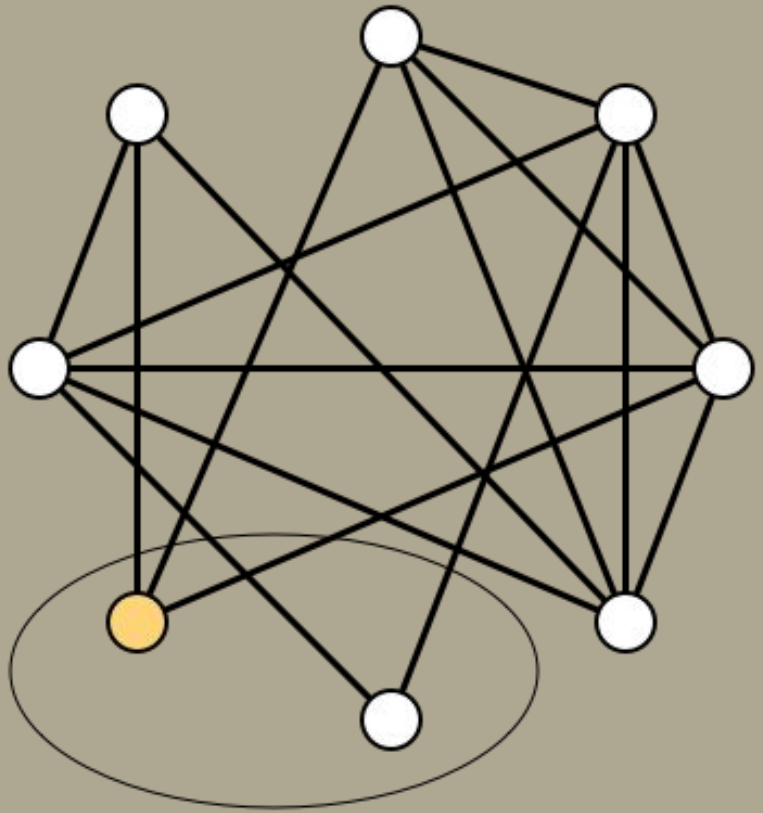
The container method gives a collection of containers $C \in P(V)$ such that:

1. There are not too many containers
2. Each independent set is fully contained in a container
3. For a graph that is ε-far from bipartite, each container is smaller than $(1-\varepsilon)n$

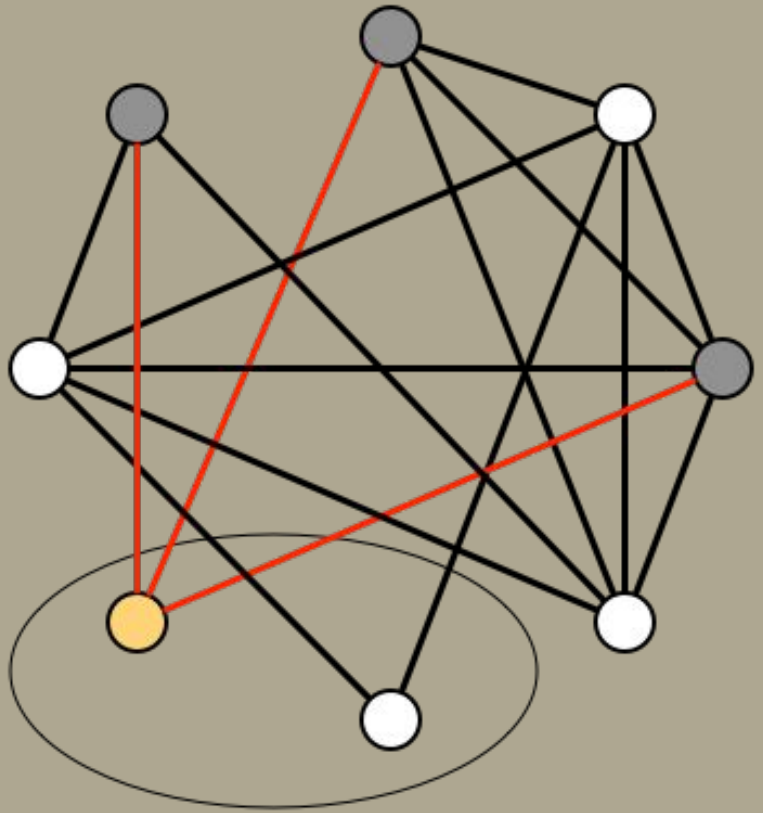
The Graph Container Method



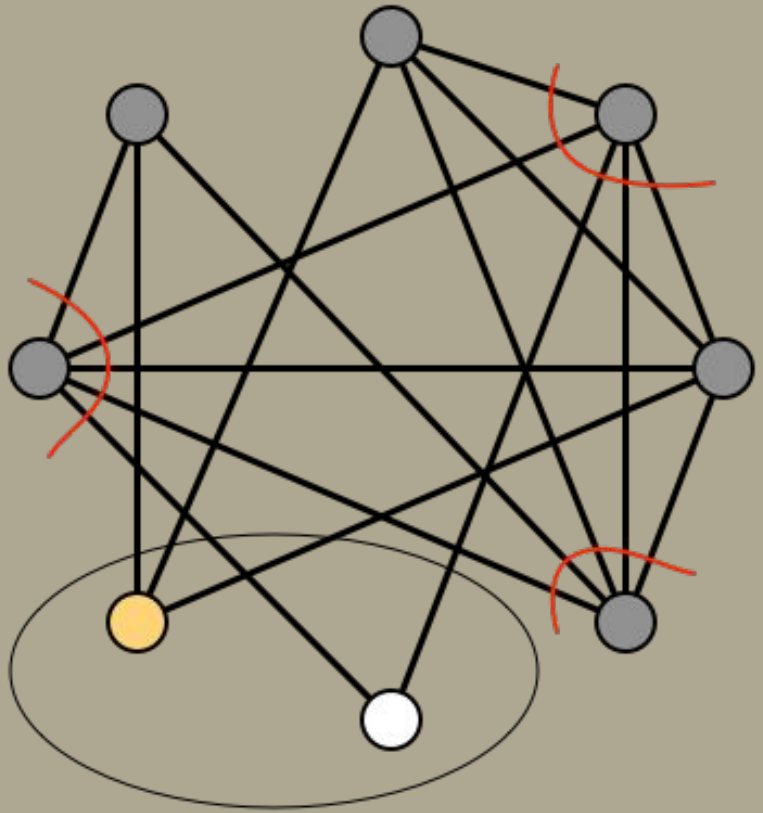
The Graph Container Method



The Graph Container Method



The Graph Container Method



We've been able to use the container method to prove Bogdanov/Li's conjecture in the cases that the containers are all small or all large!

Thank you!