Using Artificial Potential Field for Tracking of a Swarm of Drones

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Directed Research Program

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Outline

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1. Motivation
   - Swarm Robotics
   - Artificial Potential Field (APF)
2. Lorena's Contribution
   - Novel APF Approach
   - Simulations
3. Ningya and Yurim's Contribution
   - Swarm Robotics Dynamics
   - Harmonic Potential Functions (HPFs)
   - Simulations

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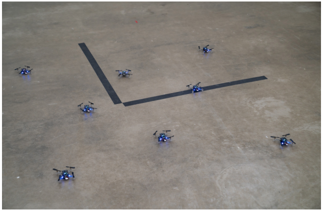
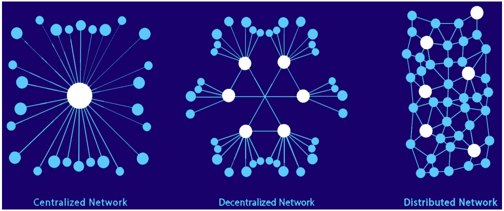
Swarm Robotics

- Coordination of multiple simple robots inspired by social insects
- **Leaderless**, **decentralized / distributed** coordination system
- Collective intelligence from **local interactions / information**
- Robust, flexible, scalable
- **Collective behaviours:** Formation / aggregation, navigation, decision making, task allocation



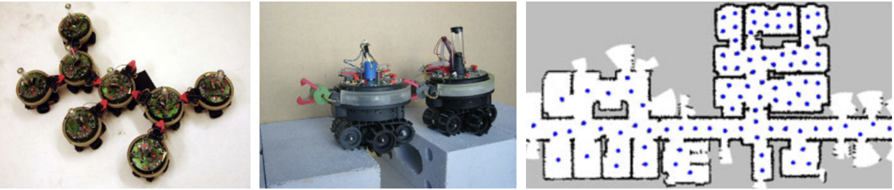
Source: [Zhang et al., 2025]



Source: Blockchain Engineer

Applications of Swarm Robotics

Application	**Swarm Properties**
Area coverage / surveillance	Distributed sensing, self-assembly
Dangerous environments	Dispensibility, robustness
Flexible large-scale systems	Scalability
Tasks requiring redundancy	Robustness



Source: [Brambilla et al., 2013]

Swarm robotics is a good framework for **multi-agent navigation**.

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Why Artificial Potential Field (APF)?

- First introduced from a paper by [Khatib, 1985]
- Each agent moves under a **virtual potential field**
- **Attractive force** pulls the robot toward the target
- **Repulsive force** pushes the robot away
  - from obstacles
  - from other robots (collision avoidance)
- Motion is determined by the **sum of forces**
- **Path planning** and **obstacle avoidance** only with local information
- Simple framework suitable for **distributed multi-agent navigation**

Traditional APF (T-APF)

In T-APF, the robot is a point mass in a force field. The total potential $U(q)$ is:

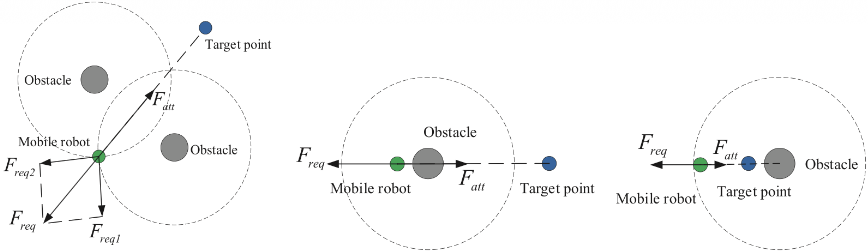
$$U(q) = U_{att}(q) + \sum_{i=1}^{n} U_{rep,i}(q)$$

The robot moves according to the negative gradient:

$$F(q) = -\nabla U(q) = F_{att}(q) + \sum_{i=1}^{n} F_{rep,i}(q)$$

Limitations of Traditional APF

- **Local minima problem**
  - Attraction and repulsion forces cancel
  - Robot may get stuck or stop before reaching the target
  - Possible oscillation at a certain position (equilibrium)
- **Unreachable goal problem**
  - Strong repulsive force near obstacles
  - Target may become unreachable



Source: Xi, Lin, Shao (2024)

Goal of Our Work

Our goal is to design a simple, stable, and efficient navigation method for multi-robot systems.

- Improve conventional **APF**-based navigation
- Achieve **smooth and stable** motion for multi-robot systems
- Maintain a **decentralized and leaderless** system

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Problem Formulation

We consider a swarm of N robots in a 2-D domain. For each robot $i = 1, \ldots, N$, the state is defined as

$$X_i(t) = \begin{bmatrix} x_i(t) \\ y_i(t) \end{bmatrix} \in \mathbb{R}^2.$$

The dynamical equation for an agent is given as

$$\dot{X}_i(t) = F_i(t),$$

where, $F_i \in \mathbb{R}^2$ is the force acting on robot i generated by APF.

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Proposed Novel Attractive Function

Instead of modifying the repulsive force, we propose a **PA-APF** using a tuning parameter D:

New Attractive Potential Equation [Azzabi and Nouri, 2019]

$$U_{att}(q) = \frac{1}{2}\zeta\left[d^2(q, q_{goal}) + \left(\frac{1}{D}\right)^2 - \left(\frac{1}{D + d(q, q_{goal})}\right)^2\right]$$

- q: Position of the robot.
- q_{goal}: Position of the target.
- ζ: How strong the attraction to the target is; positive constant.
- D: Controls the influence of the target at close range.

From T-APF to PA-APF

Traditional (T-APF) Limitation

$$U_{att}(q) = \frac{1}{2}\zeta d^2(q, q_{goal})$$

Proposed (PA-APF) Innovation

$$U_{att}(q) = \frac{1}{2}\zeta\left[d^2 + \left(\frac{1}{D}\right)^2 - \left(\frac{1}{D+d}\right)^2\right]$$

Outline

Parameters

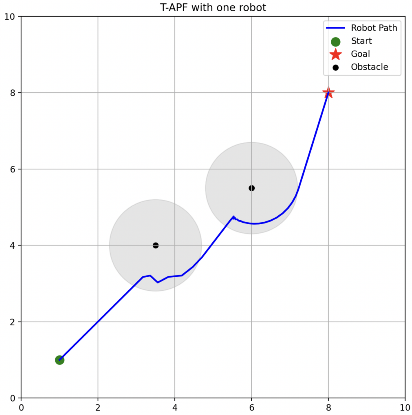
From the model formulation, we have

$$X_i^{n+1} = X_i^n + F_i \, \Delta t$$

where we set

- $n = 1200$: discrete time step (120s in 0.1s intervals),
- $X_i(t) \in [0, 20] \times [0, 20]$: domain,
- $F_i \in \mathbb{R}^2$: force acting on robot i from APF

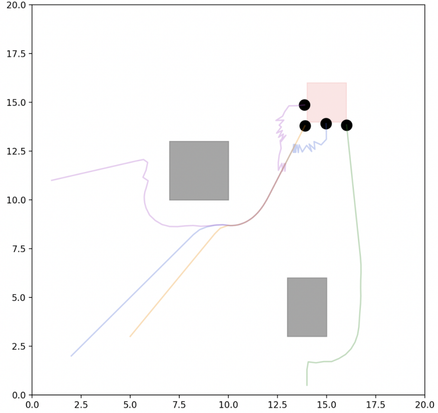
Simulation Case A: Single Robot Baseline



Setup: One robot, two static obstacles.

- Demonstrates basic obstacle avoidance using T-APF.
- The trajectory is smooth when no other agents are present.
- Establishes the baseline performance of the potential field.

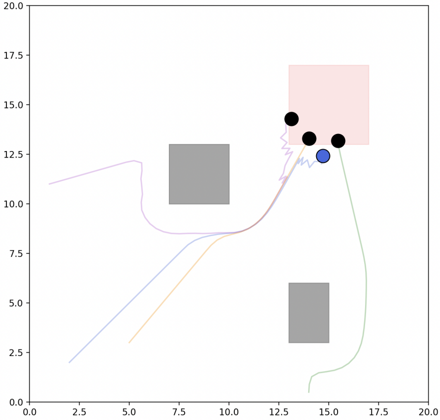
Simulation Case B: Multi-Agent Edge Attraction



Setup: Swarm attracted to target edges.

- Robots aim for the closest points on the target boundary.
- Reduces "congestion" at the center.
- Results in a distributed parking formation along the perimeter.

Simulation Case C: Center Point Convergence



Setup: Swarm attracted to target center.

- All agents prioritize reaching the exact coordinates $(15, 15)$.
- Highlights the "GNRON" and collision challenges.

Outline

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Swarm Robot Dynamical System

We consider a swarm of N robots in a 2-D domain. For each robot $i = 1, \ldots, N$, the state is defined as

$$X_i(t) = \begin{bmatrix} x_i(t) \\ y_i(t) \end{bmatrix} \in \mathbb{R}^2.$$

The continuous-time stochastic dynamics are given by

$$dX_i(t) = (F_i + u_i)\, dt + \sigma\, dW_i(t)$$

where

- $u_i \in \mathbb{R}^2$: control input obtained from the optimal control problem,
- $F_i \in \mathbb{R}^2$: force acting on robot i from APF,
- $dW_i(t) \in \mathbb{R}^2$: Brownian motion,
- σ: diffusion constant.

Methodology

After discretization, we have

$$X_i^{n+1} = X_i^n + (u_i + F_i)\Delta t + \sigma dW_i^n$$

where

- n: discrete time step,
- $X_i(t) \in [x_{min}, x_{max}] \times [y_{min}, y_{max}] \in \mathbb{R}^2$: domain.

Optimizing the control input at every step slows down the simulation for multi-robot systems.

Therefore, in this work we set

$$u_i = 0.$$

Motion is driven only by the force term F_i^n, computed from **Harmonic Potential Functions (HPFs)**.

Outline

Main Idea: Harmonic Potential Functions (HPFs)

- HPFs are functions that satisfy the Laplace equation, which are second order partial derivatives that sum to zero
- The Laplace equation's advantage is having **no local minima**
- We used the key equations below, which are from a paper by [Cetin et al., 2013]

Calculation of total gradient in two dimensions

$$d_x = \frac{2(x - x_a)}{(x - x_a)^2 + \gamma(y - y_a)^2} \qquad d_y = \frac{2\gamma(y - y_a)}{(x - x_a)^2 + \gamma(y - y_a)^2}$$

These equations are derived from:

$$-\log\left(\alpha\left((x - x_a)^2 + \gamma(y - y_a)^2\right)\right)$$

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Parameters

From the model formulation, we have

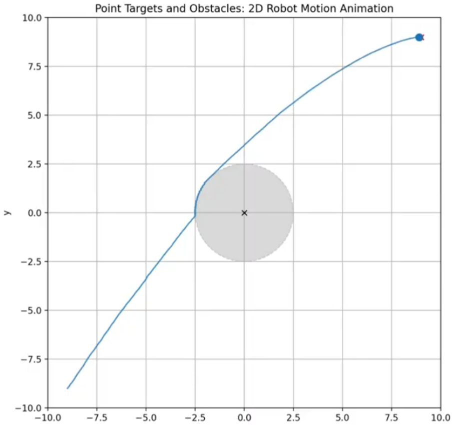
$$X_i^{n+1} = X_i^n + F_i \Delta t + \sigma dW_i^n$$

where we set

- $n = 300$: discrete time step (30s in 0.1s intervals),
- $X_i(t) \in [-10, 10] \times [-10, 10]$: domain,
- $F_i \in \mathbb{R}^2$: force acting on robot i from APF (calculated using HPFs),
  - γ: scaling parameter (adjusted between 1 and 2)
- $dW_i(t) \in \mathbb{R}^2$: Brownian motion (randomly generated),
- $\sigma = 0.003$: diffusion constant.

Simple Problem

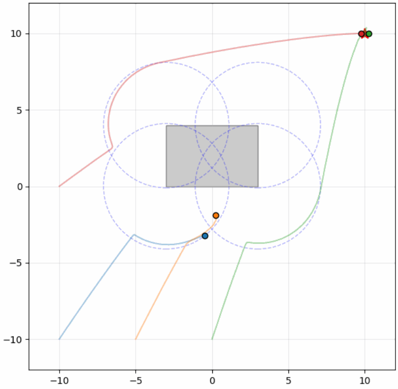
- We began with a simple system as described in the formulation



Visualization of converged system (point obstacle and target with one robot)

Challenges: Stalling and stuck robots

- HPFs are theoretically free of local minima, but the way we structured the four-point obstacle forms local minima since the repulsive forces each corner converge and cancel out

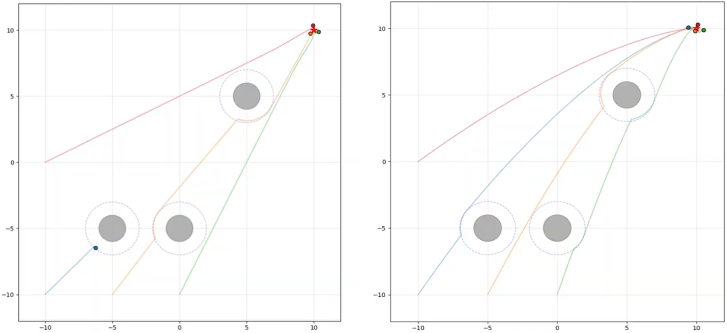


Visualization of stuck system (rectangular obstacle and point target with four robots)

Challenges: Stalling and stuck robots

- Initially, we tried an "escape force" from a paper by [Zheng, 2024]

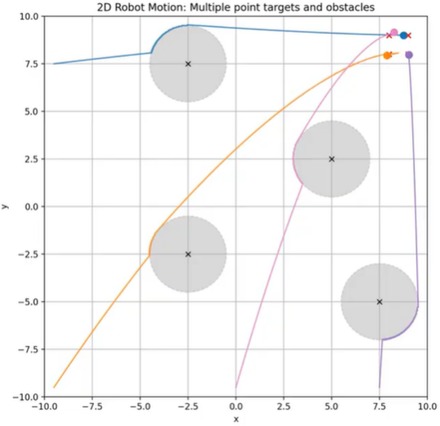
Visualization of two systems (point obstacles and target with four robots)



Stuck system (left) and converged system (right)

Challenges: Stalling and stuck robots

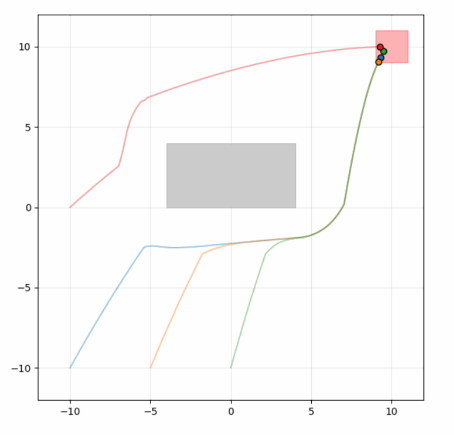
- Ultimately, we adjusted the scaling parameter (γ) to steepen the attraction gradient relative to the repulsion gradient



Visualization of converged system (point obstacles and targets with four robots)

Challenges: Colliding robots at target

- In our system, robots become stationary after they reach the target (zero forces)
- In some scenarios, this meant incoming robots would collide with arrived robots

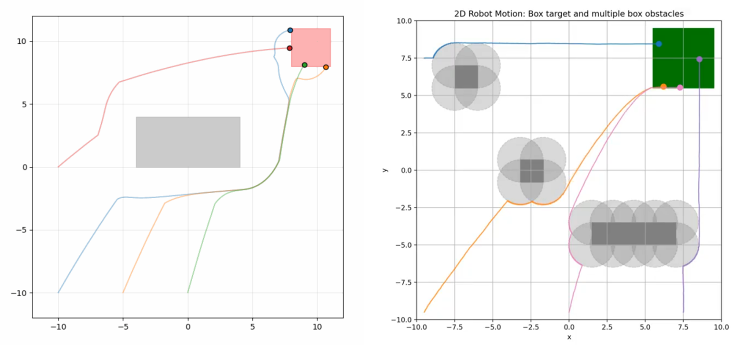


Visualization of overlapping robots (rectangular obstacle and target area with four robots)

Final Result

- To avoid robot collisions at and near the target, we continued to generate repulsive forces from the arrived robots as if they were a stationary obstacle

Visualization of two converged systems (rectangular obstacles and target area with four robots)



Key characteristics: Collision avoidance at the target for robots with overlapping paths (left)
Multiple obstacles of varying sizes (right)

Conclusions

- Lorena's contribution
  - Implementation of the PA-APF approach, and observed improved conversion compared to T-APF.
- Lorena's future work
  - Introduce noise to ensure the navigation method is realistic when faced with unpredictable disturbances.

- Ningya and Yurim's contribution
  - Implementation of harmonic potential functions in APF, resulting in improved results compared to traditional APF.
- Ningya and Yurim's future work
  - Expand the 2D system to a 3D space, and include optimization for the control input.

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Thank you for listening!