Introduction to Bayesian Inference

Keeley Isinghood and Haiyan Zhu

Mentor Yiran Wang
Department of Statistics and Actuarial Science
University of Waterloo

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Basics of Bayesian Inference

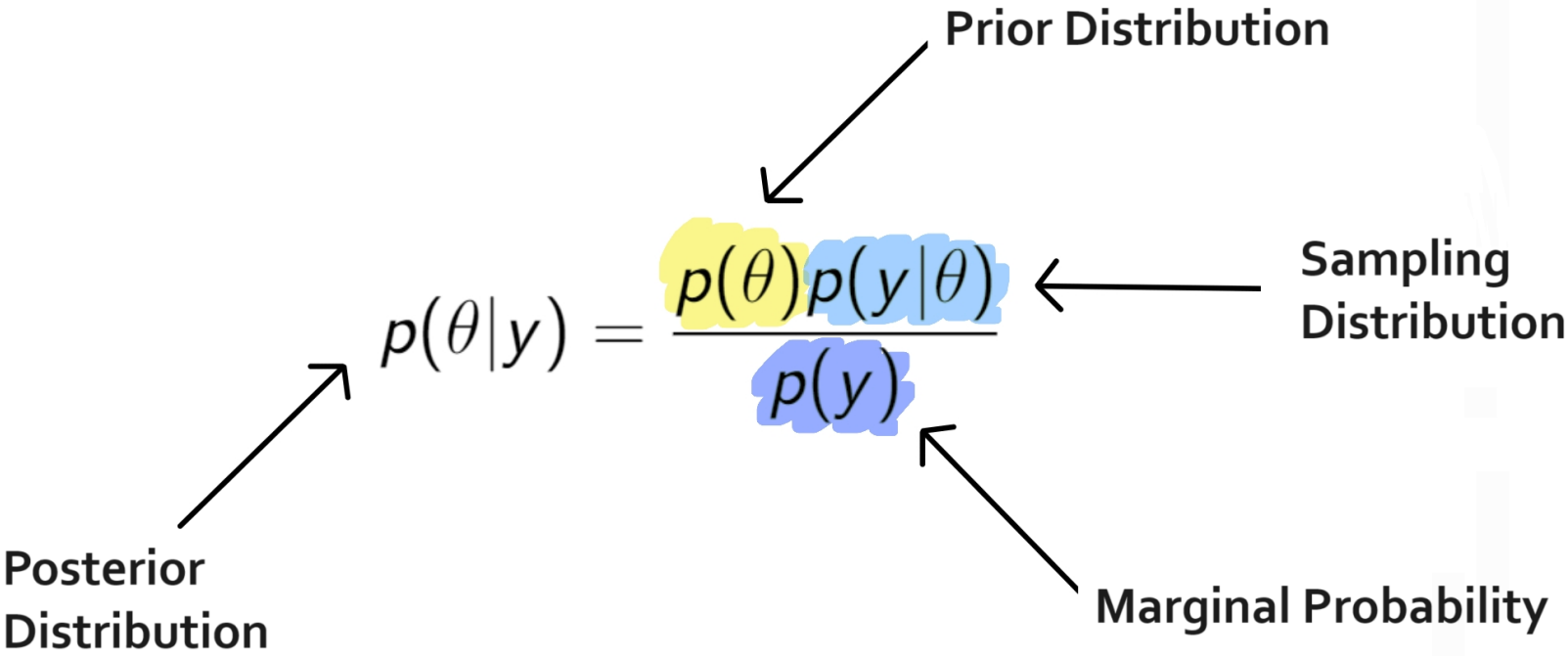
What is Bayesian Inference

- trying to find an unknown parameter based on data from an assumed distribution
- assumes that this parameter is also a random variable, with its own distribution
- Utilizes Bayes Rule

Bayes Rule

$$p(\theta|y) = \frac{p(\theta)p(y|\theta)}{p(y)}$$

A Closer Look at Bayes Rule



Challenges and How to Deal with Them

- Determining a Prior Distribution
- Computing the Integral in the Denominator

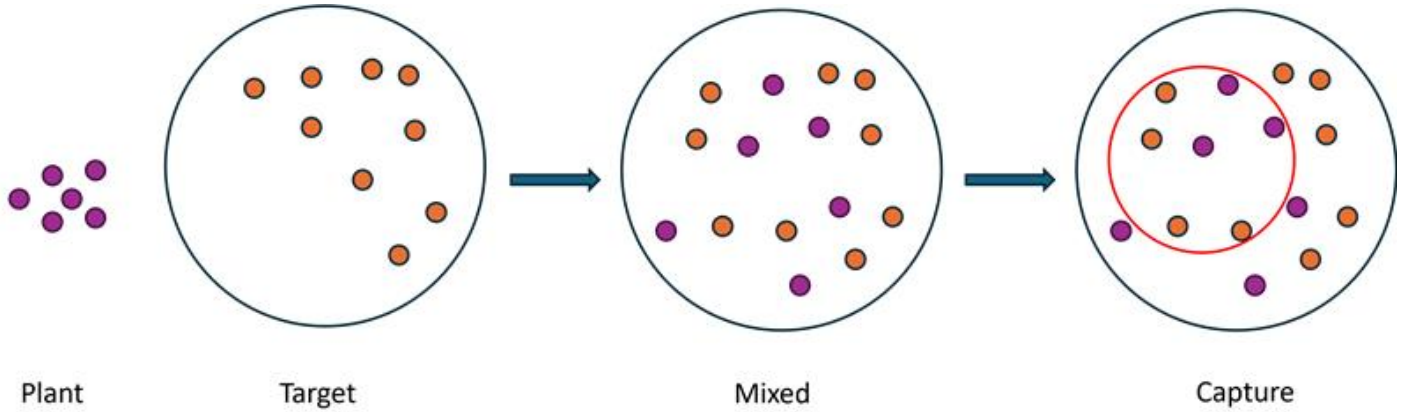
Challenges and How to Deal with Them

- Determining a Prior Distribution
  - Utilize existing knowledge or other data sources
  - Choose non-informative or weak priors to have little influence over the outcome
  - Use 'conjugate priors' which make computation easy
- Computing the Integral in the Denominator
  - Utilize computational methods and programs made for Bayesian Inference
  - MCMC (Markov Chain Monte Carlo)

Applications of Bayesian Inference

Estimating Population Size of Street-Dwelling Homeless

- Plant-capture studies:



- Challenges: No plant-capture studies have been conducted in Edmonton.
  - Uncertainty in population size.
  - Costs and optics of compensating individuals to pretend to be homeless.

Estimating Population Size of Street-Dwelling Homeless

- Strategy: Use plant-capture data from Toronto; construct prior distributions and update to obtain posterior distribution.

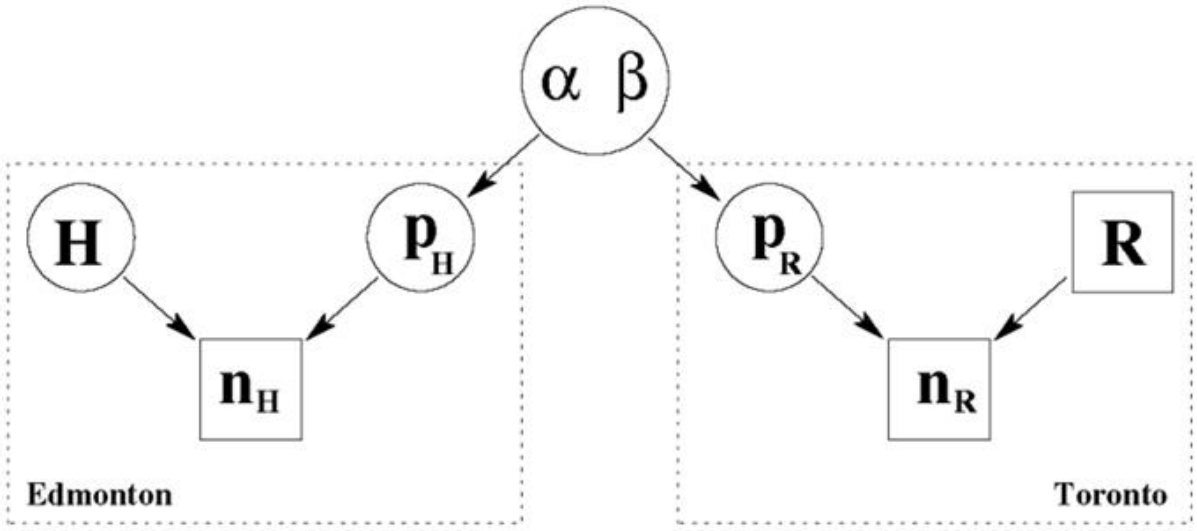


Figure 3. Probabilistic graphical model showing the conditional independence structure between data and unknown parameters in Edmonton and Toronto. Square boxes indicate quantities that are fixed and known, circles indicate unknown quantities. Our objective is to estimate $\mathbf{H} = (H_{1999}, \ldots, H_{2012})$, the size of the homeless population in Edmonton for each year

- Further applications: Can be used to assess homeless counts in other cities.

Bayesian Forecasting in Macroeconomics

- How to forecast?
  - Gather extensive time series data on macroeconomic indicators.
  - Display significant co-movements.
  - Require joint modeling for accurate forecasts.
  - Challenge: Large-scale models are prone to overfitting and lead to weak out-of-sample forecasting performance.
- Why Bayesian methods?
  - Handle numerous parameters.
  - Address model uncertainty.

Bayesian Forecasting in Finance

- How to forecast?
  - Determine if risks associated with financial assets and prices are predictable.
  - Predict latent distributional features and develop complex, non-linear state-space models
- Why Bayesian methods?
  - Produce predictive distributions automatically.
  - Provide computational flexibility.
  - Facilitate estimation and forecasting of complex models, primarily through MCMC algorithms.

When does Bayesian Inference not work?

Possible Drawbacks of Bayesian Statistics

- Not all variables can be modeled, but this is an assumption
- Subjectivity in selecting a prior distribution

Considerations to Make it Work

- Consider what variables make sense to be a part of the model and what data you have
- If you make assumptions or don't include something in the model this will limit the conclusions you can draw
- The more data the better, importance of large sample size
- Never think of the modelling process as static, check whether your results makes sense and try again
- Be wary of statistical inference produced using any methods

References

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Thank You