Applying robust optimization to the green vehicle routing problem

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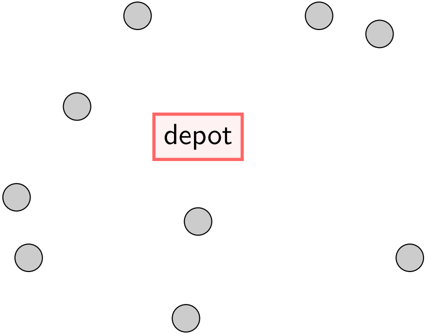
Directed reading program - Women in math

Mentor: Matheus Jun Ota

May 9, 2024

Capacitated vehicle routing problem (CVRP)



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N : customers

0 : depot

k : number of vehicles

c_a : arc cost

q_i : demand of i

Q : vehicle capacity

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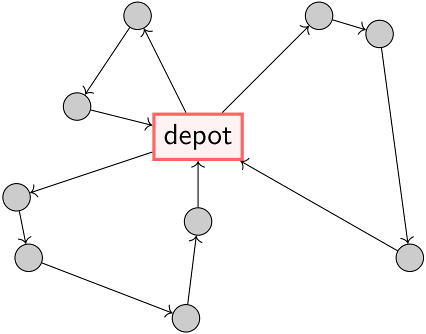

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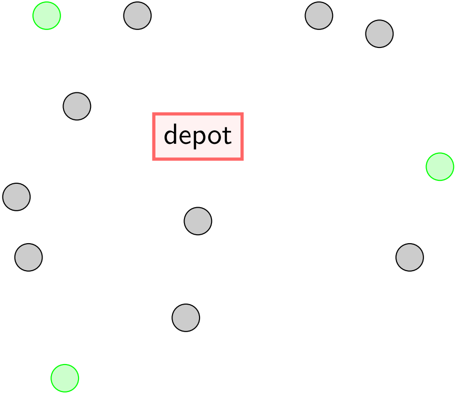

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$$N, k, c \in \mathbb{R}^A, Q, q \in \mathbb{R}^N$$

t_a : energy consumption

B : battery capacity

F : alternative fuel stations (AFS)

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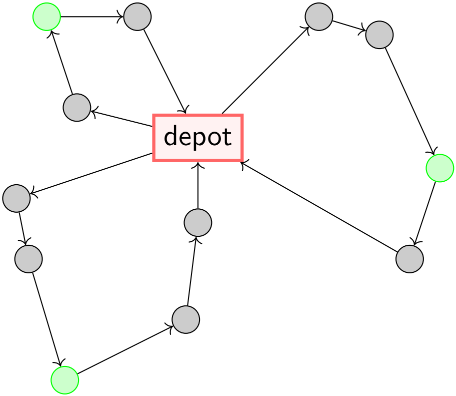

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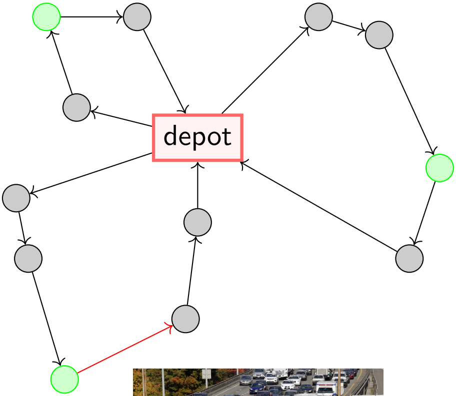

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~~t_a : energy consumption~~
t_a : random variable for energy consumption (avg. $\bar{t}_a$ and std. dev. $\tilde{t}_a$)
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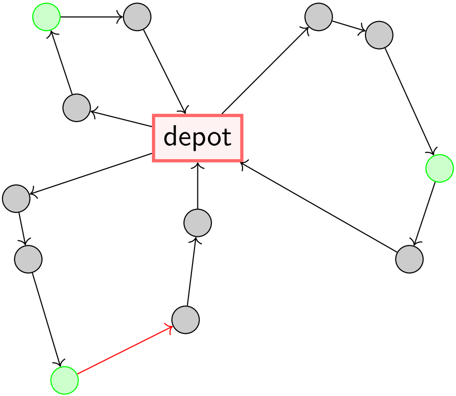

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F, B
r.v. t with params. $(\bar{t}, \tilde{t})$

Robust G-VRP (RO-G-VRP)

Let $\mathcal{T}$ be the set of all possible realizations of t. We want to ensure that we have enough energy even in the worst case realization of t.

Overview

Green logistics

Development of systems with increased energy efficiency and lower carbon emissions (Leggieri and Haouari 2017).

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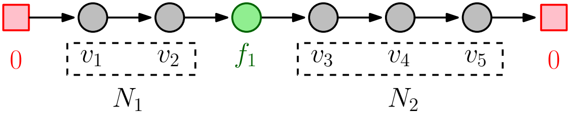
[3] "Branch-Cut-and-Price for the Robust Capacitated Vehicle Routing Problem with Knapsack Uncertainty", Pessoa et al., 2021.

Our goal: *adapt the technique of [3] to the G-VRP*[1] *and then solve the problem using VRPSolver*[2].

Mathematical formulation

A route is a sequence of customers and stations $R = (0, N_1, f_1, \ldots, N_\ell, 0)$ with total demand at most Q.



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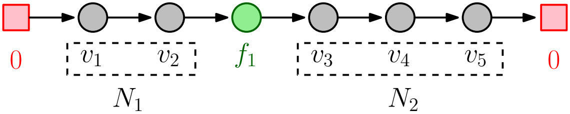

We seek k routes $R_1, \ldots, R_k$ such that

(i) for every pair $i, j \in [k]$ with $i \neq j$, we have that $V(R_i) \cap V(R_j) \cap N = \emptyset$ (i.e., no customer is visited more than once); and

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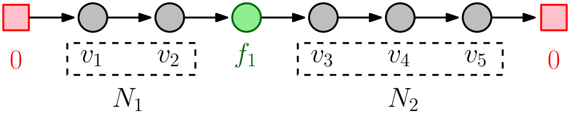

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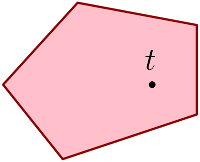
(ii) for each $i \in [k]$, R_i is energy robust feasible, i.e., for each $N_q = (v_1, \ldots, v_p) \in R_i$: $t(A(N_q)) \leq B,$ for all $t \in \mathcal{T}$

$\mathcal{T}$:



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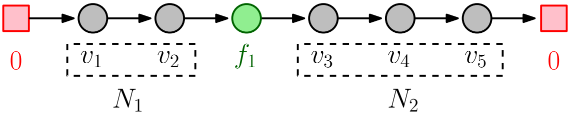

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Let $\Xi = \{\xi \in [0,1]^n : \sum_{a \in A} \xi_a \leq \Delta\}$, we use

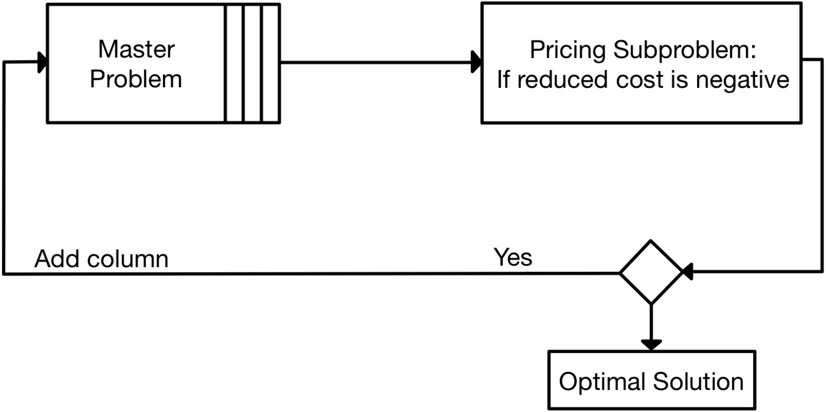
$$\mathcal{T} \coloneqq \{\bar{t} + (\tilde{t})^\top \xi : \xi \in \Xi\}.$$

Set-Partitioning Formulation

Let $\mathcal{R}$ be the set of feasible routes. Let λ_R be a decision variable of route $R \in \mathcal{R}$. We use the following set partitioning formulation.

$$
\begin{aligned}
\text{(RO)} \quad & \min \quad \sum_{R \in \mathcal{R}} c(R)\lambda_R && (1) \\
& \text{s.t.} \quad \sum_{R \in \mathcal{R}} \text{COUNT}(v, R)\lambda_R = 1, \qquad \forall v \in N, && (2) \\
& \qquad \sum_{R \in \mathcal{R}} \lambda_R = k, && (3) \\
& \qquad \lambda \text{ binary.} && (4)
\end{aligned}
$$

Column Generation and Pricing Subproblem



Energy robust feasibility constraint

Fix $R = (0, N_1, f_1, \ldots, N_\ell, 0) \in \mathcal{R}$ and let $N_i \in R$.
Let $z \in \{0,1\}^A$ be s.t. $z_a = 1$ iff $a \in A(N_i)$. (z is fixed, so it is not a variable!)

$$\max_{t \in \mathcal{T}} \left\{ \sum_{a \in A} t_a z_a \right\} \leq B \tag{5}$$

Constraint (5) seems **hard** to handle.

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Constraint (5) seems **hard** to handle. We will use the technique of (Pessoa et al., 2021) [3] to rewrite (5) as

$$\exists \theta \in \Theta : \sum_{a \in a} t_a^\theta z_a \leq B^\theta. \tag{6}$$

Constraint (6) is **easier** to handle and we can model it inside VRPSolver. For each N_i, we need to choose $\theta \in \Theta$ such that $t^\theta(N_i) \leq B^\theta$.

VRP Solver

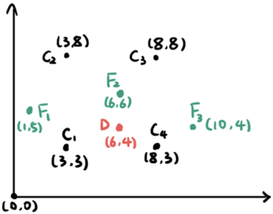
```
NAME : A-v8-car4
TYPE : RO-GVRP
DIMENSION : 8
EDGE_WEIGHT_TYPE : EUC_2D
ENERGY_CAPACITY : 10
WEIGHT_CAPACITY: 2
NODE_COORD_SECTION
1 6 4
2 3 3
3 3 8
4 8 8
5 8 3
6 1 5
7 6 6
8 10 4
PRICE_SECTION
6 2
7 3
8 1
DEMAND_SECTION
```

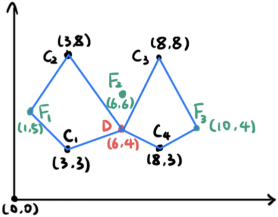
```
Solution:
1->2->6(charge 5)->3->1
1->5->8(charge 5)->4->1
```

```
# Formulation
rogvrp = VrpModel()
@variable(rogvrp.formulation, x[id in edge_ids], Int)
@objective(rogvrp.formulation,
    Min, sum(get_energy_cost(data, id) * x[id] for id in edge_ids))

for i in customers
    @constraint(rogvrp.formulation,
        sum(x[id] for id in get_out_edges(data, i)) == 1.0)
    @constraint(rogvrp.formulation,
        sum(x[id] for id in get_in_edges(data, i)) == 1.0)
end

```





Any Questions?