AM-Reading - 4

How math unveils secrets of infectious diseases like COVID-19

Mentor – Zoya Abbasi
Mentees - Yashila Barnwal, Gaurika Gupta

Predicting and Controlling Disease Outbreaks

Imagine a highly infectious disease begins spreading in your community. Each day, new cases emerge, overwhelming healthcare systems and sparking fear among the population.

What do we do next?

- How can we understand the spread of the disease?
- What strategies can we design to control it?
- How can we protect vulnerable populations and minimize the impact?

Risk factors

Age

Many diseases occur more frequently in certain age brackets

Genetics

Some diseases have a hereditary component and can be passed down through families

Lifestyle

Certain behaviors can increase the risk of developing certain illnesses

Environment

Exposure to certain substances in the environment can increase the risk of developing some diseases

Medical history

A personal or family history of certain illnesses can increase the risk of developing related or similar conditions

Gender

Some illnesses are more common in one gender than the other

Modelling the population

SIR Epidemic Model

- Susceptible
- Infected
- Recovered

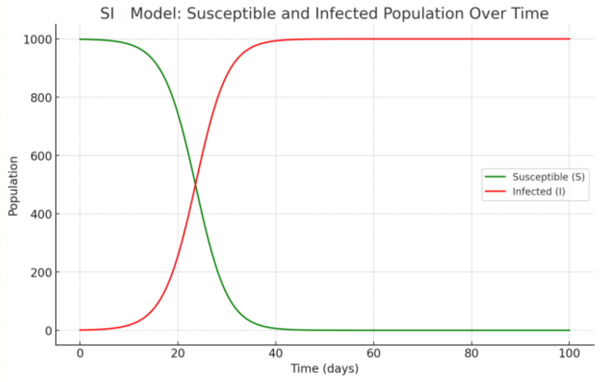
Who is at risk?

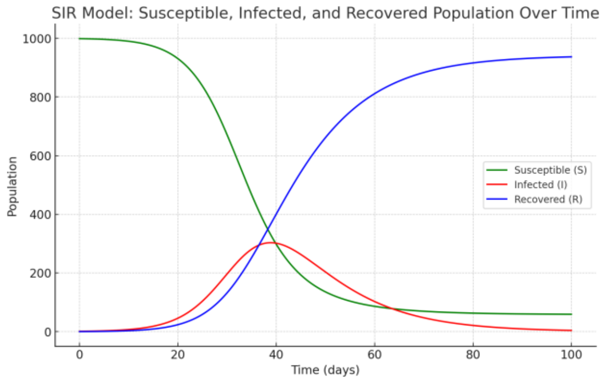
Susceptible → **Infected** → **Recovered**

$$\frac{dS}{dt} = -\beta S I$$

$$\frac{dI}{dt} = \beta S I - g I$$

$$\frac{dR}{dt} = g I$$





Definitions

β: Contacts per infected person per day

g: Recoveries per person per day

$\frac{1}{g}$: Number of days it takes to recover

$\frac{\beta}{g}$: Contacts per infection

c : Contact rate

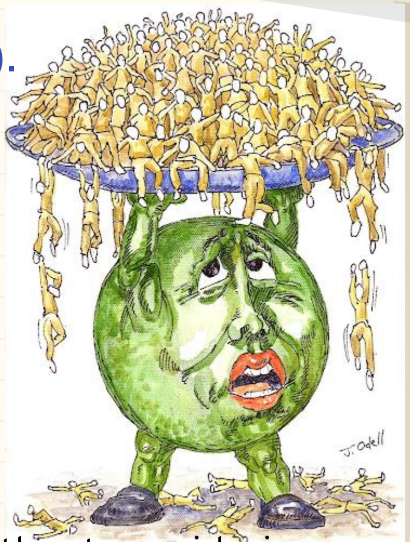
p : Probability of transmission

$\beta = pc$ (we assume $p = 1$)

Total Population (N) Formula

The entire population, $\boldsymbol{N}$, is split into three groups:

Susceptible ($\boldsymbol{S}$) , Infected ($\boldsymbol{I}$) , and Recovered ($\boldsymbol{R}$).



$$N = S + I + R$$

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt}$$
$$= -\beta SI + \beta SI - gI + gI$$
$$= 0$$

For simplicity, this model assumes a constant population size, N, without considering births, deaths, immigration, or growth.

Basic Reproduction Number R_0

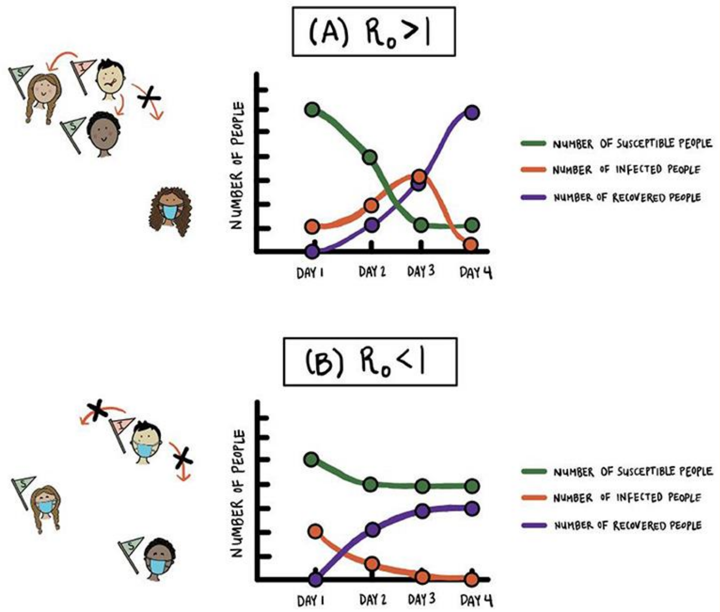
$$R_0 = \frac{\beta}{g}$$

- $R_0 > 1$

This implies $\beta > g$ so each infected individual, on average, infects more than one person, which leads to a potential epidemic because the infection can spread through the population.

- $R_0 > 1$

This implies $\beta < g$ so Each infected individual infects less than one person on average. This means the disease is not able to sustain itself in the population and will gradually die out



Strategies to Control Epidemic Spread

For the disease to be eradicated, this equation must be satisfied:

$$S(t) < \frac{g}{\beta}$$

1. Decrease β: Isolate infected people, work from home, shut down schools and colleges, and so on.

2. Decrease $S(t)$: Vaccination

3. Increase g: Antiviral treatment

4. Natural Decline: Epidemic wanes as susceptibles are exhausted

Normalization

Values are normalized to a standard scale, which is usually between 0 and 1.

$$N = S + I + R$$

$$1 = \frac{S(t)}{N} + \frac{I(t)}{N} + \frac{R(t)}{N}$$

$$1 = S(t) + I(t) + R(t)$$ (assuming N = 1)

Population Births and Deaths

In SIR model, we assume an evenly distributed birth-death process.

$\boldsymbol{\mu}$ = Birth rate = Death rate

Assumptions:

- Birth and death rates are the same value.
- All new individuals are born susceptible.
- Disease-induced deaths are not considered here.

Modified SIR Models with Births and Deaths

Susceptible Equation:

$$\dot{S}(t) = \mu - \beta I(t)S(t) - \mu S(t)$$

Infected Equation:

$$\dot{I}(t) = \beta I(t)S(t) - gI(t) - \mu I(t)$$

Recovered Equation:

$$\dot{R}(t) = gI(t) - \mu R(t)$$

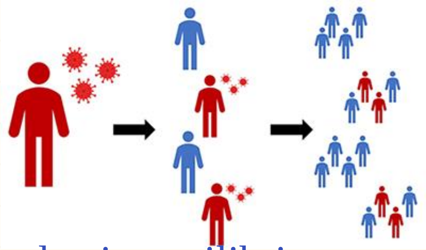
μ = Birth rate = Death rate

Equilibrium points of SIR

For a Disease-Free Equilibrium:

If $R_0 < 1$:

The disease will eventually die out, and the population will reach the disease-free equilibrium.



For an Endemic Equilibrium:

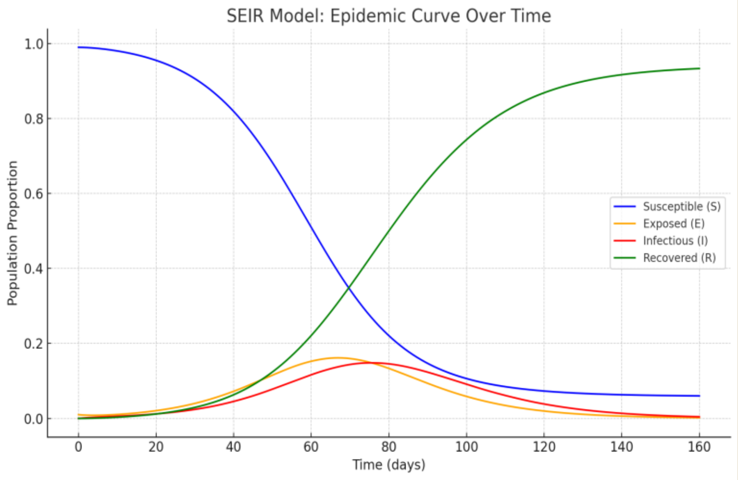
If $R_0 > 1$:

The disease will continue to exist in the population and reach an endemic equilibrium. Here, each infected person infects more than one other person on average, allowing the disease to persist at a stable level without completely disappearing or exploding into an epidemic.

Variants of the SIR Model - SEIR Model

People who have been exposed to the infection but are not yet contagious are kept in this field.

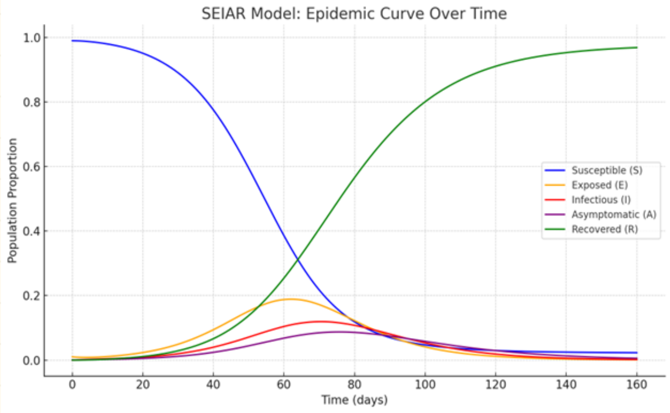
Equation	Interpretation
$\frac{ds}{dt} = -\beta SI(t)$	Indicates that susceptibility decreases with infection exposure.
$\frac{dE}{dt} = \beta SI(t) - kE(t)$	Suggests that exposed people become infected after an interval.
$\frac{dI}{dt} = kE(t) - gI(t)$	Infected individuals go toward recovery.
$\frac{dR}{dt} = gI(t)$	After recovering, they move into the recovered compartment.



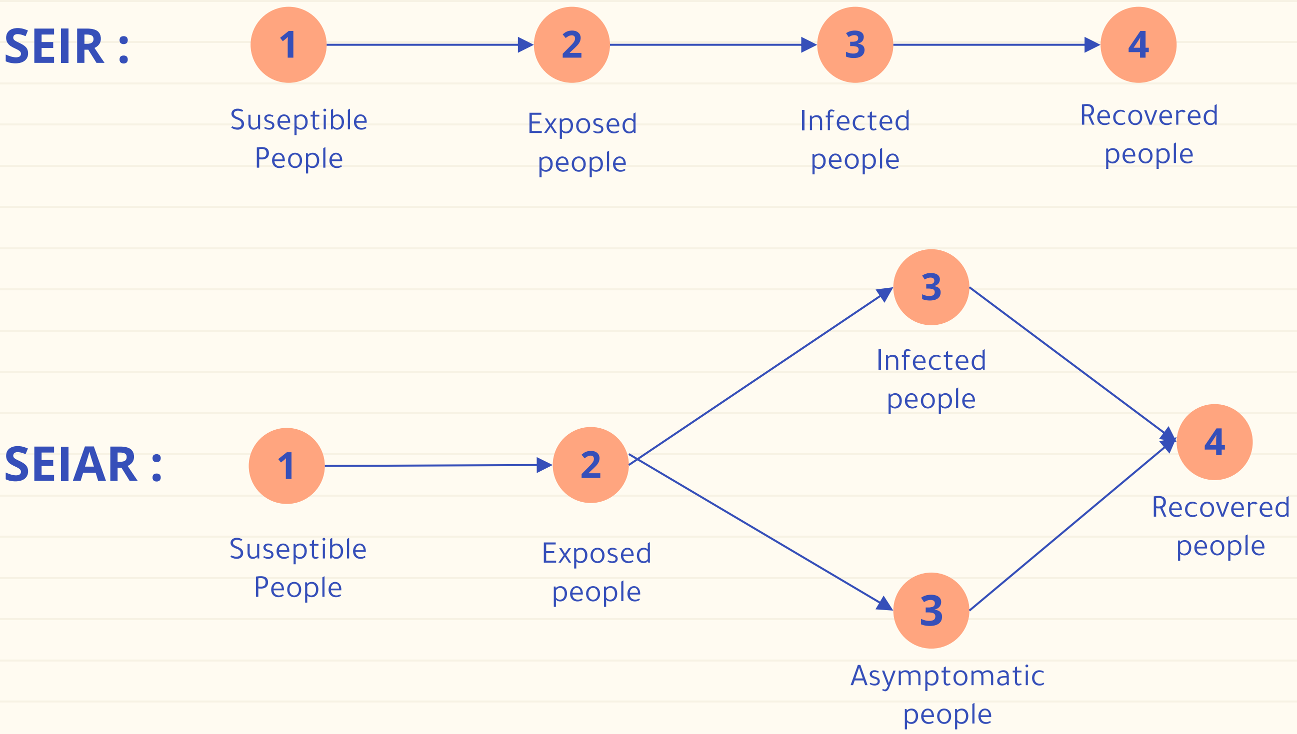
Variants of the SIR Model - SEIAR Model

Asymptomatic (A): People who are infected yet do not exhibit symptoms

Equation	Interpretation
$\dot{S}(t) = -\beta I(t)S(t)$	Indicates that susceptibility decreases with infection exposure.
$\dot{E}(t) = \beta I(t)S(t) - pkE(t) - (1-p)kE(t)$	People who are exposed either enter the infectious or asymptomatic phases.
$\dot{I}(t) = pkE(t) - gI(t)$	Infected individuals go toward recovery.
$\dot{A}(t) = (1-p)kE(t) - \eta A(t)$	Asymptomatic people either recover or continue to be asymptomatic.
$\dot{R}(t) = gI(t) + \eta A(t)$	Both asymptomatic and symptomatic cases are recovered individuals.



SEIR and SEIAR Epidemic Model



1. Martcheva, Maia. "*An Introduction to Mathematical Epidemiology*". Springer, 2015.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